Abstract

The present thesis deals with some problems associated with harmonic mappings in the open unit disc $\mathbb{D}$. We will use both the terms "harmonic function" and "harmonic mapping" to mean a complex-valued harmonic function throughout the thesis. First our research focuses on harmonic univalent mappings with nonzero pole in $\mathbb{D}$. We consider the class $\mathcal{A}_{\mathcal{H}}(p)$ of all sense preserving harmonic functions f in the open unit disc $\mathbb{D}$ having a simple pole at $z = p \in (0, 1)$ with the normalizations $f(0) = f_z(0) - 1 = 0$. We first derive a sufficient condition for univalence of such functions. We denote the univalent subclass of $\mathcal{A}_{\mathcal{H}}(p)$ with an additional normalization $f_{\bar{z}}(0) = 0$ by $\mathcal{S}^0_{\mathcal{H}}(p)$. We study the class $\mathcal{S}^0_{\mathcal{H}}(p)$ in the geometric function theoretic viewpoint. As a byproduct of our investigation, we see that consideration of nonzero pole yields nontrivial lower bounds for the Taylor coefficients of such functions. We also consider the class $\mathcal{S}^0_{\mathcal{H}}(1)$ of all sense preserving univalent harmonic mappings $f = h + \overline{g}$ in $\mathbb{D}$ having simple pole at $z = 1$ with the same normalizations as above. We obtain some coefficient bounds for the functions of certain subclass of the class $\mathcal{S}^0_{\mathcal{H}}(1)$. Further we obtain some distortion bounds for certain subclass of $\mathcal{S}^0_{\mathcal{H}}(p)$. We also prove a growth result for a non-vanishing subclass associated with $\mathcal{S}^0_{\mathcal{H}}(p)$. Thereafter, we derive the exact regions of variability for the second Taylor coefficients of h when $f = h + \overline{g}$ belongs to certain subclass of $\mathcal{S}^0_{\mathcal{H}}(p)$. Also we obtain the exact region of variability of the Taylor coefficients of h when $f = h + \overline{g} \in \mathcal{S}^0_{\mathcal{H}}(1)$ satisfies certain conditions. After this, we turn our attention to stable harmonic mappings, introduced and studied by R. Hernandez and M. J. Martin. We first produce some more results on stable harmonic mappings. Furthermore, we consider the meromorphic analogs of stable harmonic mappings and establish various results for these new classes of mappings. Lastly, we obtain some results on the neighborhood of harmonic mappings. To be more specific, we generalize some of the results of Stephan Ruscheweyh on neighborhoods of univalent analytic functions to stable harmonic mappings.

Keywords: Harmonic univalent functions, Shear construction, Taylor coefficients, Concave univalent functions, Distortion theorem, Growth theorem, Region of variability, Stable harmonic mappings, Meromorphic functions, Harmonic convolution, Neighborhoods of univalent functions, Fully starlike functions, Fully convex functions.