Abstract

This thesis presents a consolidated study on certain novel discretization models of 2D circles and ellipses, as well as 3D ellipsoids, and discusses some of its graphic applications. In this thesis, characterization of circles up to dyadic rational form on the integer plane has been thoroughly investigated, and novel techniques have been proposed for constructing circles with rational number and dyadic rational number specifications on the integer plane. Some intriguing properties of circles with rational number specifications in the integer plane have been found, which help in obtaining the minimal set of 0-connected integer points that are closest to the corresponding real circle. Due to the inherent property of dyadic rational number, construction of a digital circle with dyadic rational specifications requires much simpler basic operations. Efficient integer-based algorithms have been designed for generating both types of digital circles, and they are guaranteed to produce a correct output for any input.

Next, a slightly more challenging problem on characterization and construction of digital ellipses with two different forms of specifying the inputs have been considered, the first one being fully integers and the second being dyadic rationals. As a digital ellipse with dyadic rational number specification is likely to be asymmetric, its construction is comparatively much more challenging. It has been shown how elementary number-theoretic analysis helps in characterizing a digital ellipse through certain recurrences on the integer intervals. This, in turn, aids in designing novel ellipse drawing algorithms on the integer plane that uses only integer operations, whether the ellipse has an integer or a dyadic-rational specification.

Following this, a more challenging problem has been attempted wherein the construction of digital ellipsoid to its thinnest topological model has been solved. It has been shown how this ellipsoid model admits certain characterization based on isothetic-distance and functional-plane properties, resulting in the derivation of certain recurrences on the integer intervals that contain the values of a specific integer expression corresponding to the integer points constituting the digital ellipsoid. The proposed algorithm is the first algorithm in the literature of digital geometry for the construction

of the thinnest model of the ellipsoid in the integer space.

Following this, a technique has been proposed to solve the problem of mapping an image onto the voxelized surface of a digital ellipsoid by resolving some critical issues. A methodology using only integer operations has been introduced for flattening the voxelized surface of a digital ellipsoid by preserving the shortest path-length between two points, which harnessed the properties of digital ellipsoid. Different issues of mapping have been addressed from the perspectives of digital geometry in order to get over the gaps and unconnectedness in the unfolded ellipsoid, and by overcoming these issues, an effective mapping method has been devised.

Finally, a relevant problem in the context of designing a digital globe has been attempted to be solved. The solution is based on discretization of the earth's surface as a minimal set of voxels for the purpose of mapping a planar world map on the discretized surface. In addition, a supporting indexing technique has been developed for the designed digital globe. A scalable mapping algorithm for the digital globe has been designed that makes use of bidirectional relationships between the input planar map and the thinnest digital ellipsoid. Next, a fast indexing method by reducing three-dimensional space to one-dimensional space in order to access and operate on any of its specified voxels in constant time has been proposed.