Abstract

Spectral methods provide accurate and efficient fixed-grid approximations to problems with smooth solutions. However, in the presence of an interface, the spectral approximation of the piecewise-continuous solution exhibit the *Gibbs-Wilbraham phenomenon*, which appears as oscillation in the approximation and a drastic reduction of rate of convergence. Although, many studies have approached the problem of resolving the *Gibbs oscillation* from spectral approximation, here we present a method amenable to a wide range of physical problems as well as restore exponential convergence. We propose a reconstruction technique where the numerical solution is expressed as the sum of a smooth function and a *modified Heaviside function*. A *Chebyshev polynomial series* is used to represent the smooth function while the interface function modifying the Heaviside step is expressed in a weak form using the jump conditions to resolve the discontinuity at the interface. The solution reconstruction allows us to impose the conditions at the interface exactly. We present the rationale underlying the reconstruction by revisiting the work of Wilbraham, H. "On certain periodic functions", Cambridge Dublin Math. J 3 (1848): 198-204, followed by the numerical implementation. We introduce the weak formulation as applied to an ordinary differential equation (ODE) in one dimension with one interfacial discontinuity. We extend the methodology to solve a system of coupled ODEs governing the growth of an avascular tumor. The class of moving interface problems in phase change or the *Stefan problem* is discussed thereafter, with two different time discretizations: the *Crank-Nicolson method* and the Chebyshev reconstruction method. The weak-form based formulation is implemented to solve the *Laplace and Poisson equation* with an interface in two and three dimensions. In higher dimensions, the maximum pointwise error achieved in the entire domain is $\sim \mathcal{O}(10^{-14})$ using approximately 30 Chebyshev modes in each direction. The algorithm is also efficient, requiring less than a second of computational time for each of the problems. The pointwise error plots presented demonstrate that the Gibbs-Wilbraham phenomenon is accurately resolved. Finally, we present the correction function approach where the interface function is solved for as the solution of a partial differential equation, making it distinct from the weak-form approach. The novel approach does not rely upon the explicit integrable form of the jump condition, unlike the weak-form. The numerical formulation of the correction function method is optimized for accuracy as well as computational cost in two dimensions, costing a fraction of a second on a desktop computer. Concluding thoughts on the potential applications of the reconstruction technique are presented at the end.

Keywords: Chebyshev polynomials; collocation method; solution reconstruction; Gibbs-Wilbraham phenomenon; exponential accuracy