CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

1.1. Hydrodynamic and Hydromagnetic Stability

The theory of hydrodynamic stability deals in predicting transitions between laminar and turbulent configuration of a given flow field or predicting whether or not a given flow pattern is stable or unstable, and thus whether or not it can be observed in nature. The naturally occurring configuration is a consequence of the stability of physical system. These flow systems satisfy the equation of motion. In spite of complexity of the flow, there can be a stationary solution for some of the flows. The given flow field may be stable or unstable to infinitesimal or finite amplitude disturbances. Linear stability theory deals with infinitesimal disturbances, the non linear combination of disturbances are neglected, while in nonlinear stability theory these are retained.

Stability analysis of hydrodynamic or hydromagnetic systems can be described by a set of nondimensional parameters. The numerical values of these parameters will give the criterion of stability. The parameter space can be divided into stable and unstable zones. In the stable zone, any arbitrary disturbance to the system must tend asymptotically to zero as time tends to infinity. In the unstable zone there must be at least one mode of disturbance which grows with time. The locus of points which separates the stable zone from the unstable zone is known as the state of marginal stability of the system. State of marginal stability can be of two types. In the first type, transition from stability to instability takes place via a marginal state exhibiting a stationary pattern of motions. In this case the marginal state is said to be stationary and the principle of exchange of stabilities is said to be valid. In the second type, the transition from stability to instability takes place via a marginal state exhibiting oscillatory motion with a certain definite characteristic frequency and is referred to as overstability (Chandrasekhar, 1981).

In the thesis we have explained two types of hydrodynamic stability problems. Dynamic stability is one of them where stability of the system is achieved by modulating any one of the operating parameters. Simple example is inverted pendulum which gets stabilized by oscillating the point of support. Another example of dynamic stabilization is ion trap. In case of an ion trap it requires minimum potential which can be obtained by properly modulating electrode potential (Paul, 1990). Both the methods mentioned above are single degree freedom systems whereas dynamic stabilization can also be applied for multi degree freedom systems. This was explained by Berge (1972) in one of his review paper where a plasma column was confined by an oscillating magnetic field. Similar dynamic stabilization for multi degree freedom system has been shown in case of Rayléigh–Taylor instability of superimposed liquids by oscillating the liquids perpendicular to the interface (Wesson, 1970; Wolf, 1970). Dynamic stabilization has practical importance in plasma confinement by oscillating magnetic field. Another application of dynamic stabilization is found in material processing industry, where modulated temperatures are imposed on a desired temperature difference across the boundaries to delay the onset of convection. Other set of problems deals with analysis of different fluid flow configurations where the whole system is divided into two regions either due to geometry or due to presence of two different fluids because of which the stability is governed by local Taylor number. This type of situation is observed in mechanical seals where there is step change in radius and in emulsification process where two liquids superimposed one upon the other.

The review is far from an exhaustive one. But emphasis has been put on the topics and methods directly relevant to the present dissertation.

1.2. Centrifugal Instability

Onset of instability can occur under variety of externally impressed conditions. A potentially unstable arrangement of flow in the annulus between two rotating coaxial cylinders occurs due to adverse gradient of angular momentum. This class of problems is known as Taylor-Couette flow, after the two pioneering researchers in the field, Taylor (1923) and Couette (1890). The flow configuration is shown in Fig.1.1.

The linear stability of a steady non-dissipative flow of an incompressible fluid with circular streamlines between two coaxial cylinders was first investigated by Rayleigh (1917), for axisymmetric disturbances, who showed that a stratification of angular momentum about an axis is stable if and only if it increases monotonically outward. This necessary and sufficient condition for stability is known as Rayleigh’s criterion. Taylor (1923) extended the analysis of Rayleigh (1917) to include the effect of

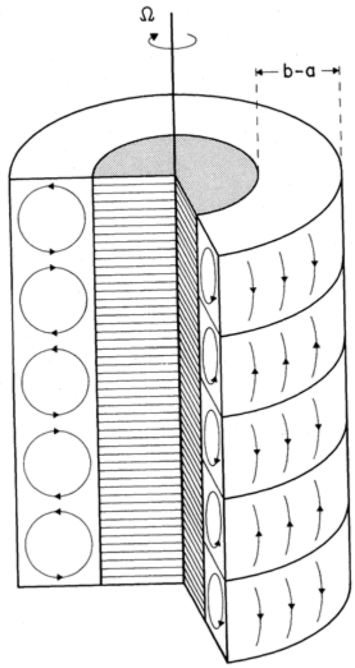

Fig.1.1: Schematic picture of Taylor-Couette system, showing fluid streamlines in Taylor-vortex state when the inner cylinder is rotating (Cross and Hohenberg, 1993)

viscosity and showed that viscosity has a stabilizing influence on the flow. He showed that, in this case, the stability of the flow is determined by the numerical value of a non-dimensional parameter, referred to as Taylor number, which gives a measure of the extent to which Rayleigh's criterion is violated. He calculated, under certain simplifying assumptions, the numerical value of the critical Taylor number at which instability first sets in as a regular pattern of horizontal, toroidal vortices along the length of the cylinders. Experimentally he has shown that the marginal state is stationary. The new flow is called a Taylor-vortex flow. The theoretical results obtained by Taylor were in excellent agreement with his experimental observations.

Taylor's original theoretical investigation was limited to narrow gap only. Later stability problem of cylindrical Couette flow in wide gap was carried out by Chandrasekhar (1958), Kirchgässner (1961) and Sparrow *et al.* (1964).

The effect of a uniform axial magnetic field on the stability of cylindrical Couette flow of a viscous electrically conducting fluid was studied by Chandrasekhar (1953a). He found that the magnetic field tends to stabilize the flow. Chandrasekhar (1953a) restricted his analysis to the case of a weakly conducting fluid (low magnetic Prandtl number approximation). Kurzweg (1963) extended his analysis by considering fluids of arbitrary electrical conductivity.

So far in all these studies the cylinders are assumed to be infinite so that end effects can be ignored. The work of Benjamin (1978a, 1978b) has shown that end effects along with the aspect ratio (the ratio of the height of the cylinders to the width of the gap) play a crucial role in the formation of Taylor vortices for finite length cylinders. Finite length cylinders can have different end-plate conditions. Normally the flow over stationary end-plates is inward. For rotating end-plates (de Roquefort and Grillaud, 1978; Tavener *et al.*, 1991), the flow is outward near the rotating ends. The end-plates may have asymmetric boundary conditions, i.e., one end-plate rotating with the inner cylinder while the other end-plate is stationary (Mullin and Blohm, 2001). An odd number of Taylor vortices are formed with outward flow near the rotating end plate and inward flow near the stationary end-plate. Another possibility could be the end-plates rotate synchronously and independently of the inner cylinder (Abshagen *et al*., 2004; Moshkin *et al*., 2010).

Anomalous modes can be observed in short columns with stationary end-plates by sudden start at supercritical state. For anomalous modes, the flow is outward either at one or both the ends. Primary flow develops with gradual increase of Reynolds number from small values, for the flow to be unique (Benjamin, 1978b), and then in terms of bifurcation theory, anomalous modes are always disconnected from the primary flow and appear at much higher Reynolds numbers than the threshold for normal modes. Cliffe (1983), Lücke *et al.* (1984), Pfister *et al.* (1988), Cliffe and Mullin (1985) and Furukawa *et al*. (2002) numerically studied the formation of different anomalous modes with small aspect ratio.

The existence of Taylor vortices is not confined to the flow between straight rotating circular cylinders. They have also been observed in certain variation of the original geometry. The Taylor vortex problem between conical cylinders was investigated experimentally by Wimmer (1988, 1995) and numerically by Abboud (1988). Wimmer (1988, 1995) observed Taylor vortices of alternating large and small size. Taylor vortices can be observed between rotating spheres. The experimental study of Sawatzki and Zierep (1970) and Wimmer (1976) and numerical study by Bartels (1982) shows the formation of Taylor vortices near the equator and there is a variation in the size of the vortices as one move away from the equator. The Taylor vortex problem with eccentric cylinders was investigated theoretically by Di Prima and Stuart (1975) and experimentally by Koschmieder (1976) found the critical Taylor number increases steadily as a function of eccentricity.

However, in spite of the thorough analysis of one fluid Taylor-Couette flow, the extension to a two fluid analogue has received little attention. Theoretical work on the stability of two fluid Couette flow with inner cylinder rotation has been carried out by Renardy and Joseph (1985). They found that a thin layer of less viscous fluid near either cylinder is linearly stable. Bonn *et al*. (2004) and Toya and Nakamura (1997, 2004) carried out experiments to study the dynamics of two immiscible fluids between two rotating coaxial cylinders where, in the unperturbed state, the interface between the two liquids was planar and perpendicular to the axis of the cylinders. Bonn *et al*. (2004) observed that when centrifugal instability sets in the interface would climb the inner cylinder when the lower fluid had lower viscosity. Toya and Nakamura (1997, 2004) observed that at the interface, the bottom vortex in the less dense phase can co-rotate with the top vortex in the denser phase; the flow is countercurrent at the boundary between the two fluids.

Jeffreys (1928) modified the solution of Taylor (1923) by reducing the equations to a single sixth order differential equation in terms of azimuthal velocity. This was done to establish an analogy between the conditions in a layer of liquid heated from below and in a liquid between two coaxial cylinders rotating at different rates. This analogy is only valid for the linear theory with narrow gap approximation.

1.3. Thermal Instability

Systems are driven from equilibrium by some driving force such as a temperature difference, an electro potential difference, or a chemical reaction. For every driving force, there are dissipative mechanisms like viscosity, thermal diffusivity which act to restore the system back to equilibrium. The balance between these driving force and dissipative mechanism causes the systems to form patterns, such as stripes, squares, hexagons, and spirals. One of the best examples of a pattern-forming system is Rayleigh-Bénard convection (Bodenschatz *et al*., 2000) as shown in Fig.1.2, in which a horizontal fluid layer heated from below is subjected to a temperature difference leading to a buoyancy-driven instability.

The problem of the onset of thermal instability in a horizontal layer of fluid, heated from below in a field of gravity was examined theoretically by Rayleigh (1916). His analysis was confined to the case of two free boundaries. He showed that the stability of the layer is determined by the numerical value of a non dimensional

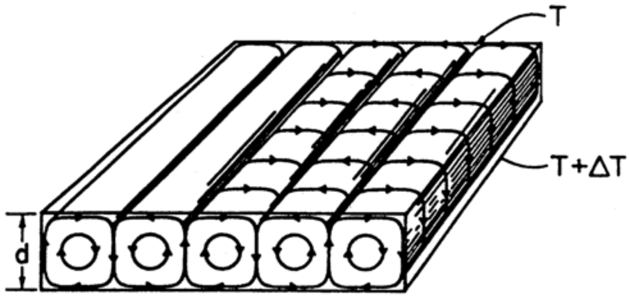

Fig.1.2: Schematic picture of Rayleigh-Bénard convection showing fluid streamlines in an ideal roll state (Cross and Hohenberg, 1993)

number referred to as Rayleigh number, which depends on the temperature gradient, the depth of the layer, the acceleration due to gravity and on the coefficient of thermal expansion, the thermal diffusivity and the kinematic viscosity of the fluid. If the Rayleigh number is below a certain value, the layer is stable. Instabilities set in as soon as the Rayleigh number exceeds this value. He also proved that the marginal state is stationary. Later Jeffreys (1928) and Pellew and Southwell (1940) extended Rayleigh's analysis to include other types of boundary conditions and proved that even in these cases the principle of exchange of stabilities is valid. Pellew and Southwell (1940) also obtained a variational principle for this problem and used it for approximate calculation of critical Rayleigh number. Chandrasekhar (1953b) and Chandrasekhar and Elbert (1955) extended Rayleigh's problem to include the effect of rigid rotation. They obtained a variational principle for this problem and calculated approximate values of the critical Rayleigh number as a function of the Taylor number (a non dimensional parameter which gives a measure of the rotation rate). They showed that in this case the marginal state may be oscillatory particularly for fluids with low Prandtl number.

1.4. Parametric Instability

Stability of modulated basic state has received relatively little attention in comparison to the stability of unmodulated basic state because of mathematical difficulties. The modulation of the basic state is important since an imposed modulation may destabilize (stabilize) an otherwise stable (unstable) state. A modulated basic state is stable if an applied perturbation decays at every instant. The simplest example is the classical inverted pendulum, which can be stabilized by oscillation of the point of support, for suitable values of amplitude and frequency of oscillation. This change of state due to oscillations is called parametric resonance or parametric instability. The parametric instability occurs due to a modulation of some parameter in a dynamical system.

Theoretical as well as experimental work has been done on the stability of hydrodynamic and hydromagnetic flow in which the basic state is oscillatory. The stability of viscous Taylor-Couette flow, when the rotation rate of the inner cylinder is modulated sinusoidally, was investigated experimentally by Donnelly (1964). He showed that the onset of instability can be inhibited by modulation. The theoretical analysis for this problem was carried out by Hall (1975), Riley and Laurence (1976), Carmi and Tustaniwskyj (1981), Tustaniwskyj and Carmi (1980), Barenghi and Jones (1989), Kuhlmann *et al*. (1989) and Barenghi (1991). They found that modulation of inner cylinder destabilizes the flow. In another investigation of time modulated Taylor-Couette flow with inner cylinder rotating at constant angular velocity and outer cylinder rotation modulated with zero mean rotation has been studied experimentally by Walsh and Donnelly (1988) and theoretically by Murray *et al.* (1990) and Lopez and Marques (2003). They observe that modulation of the outer cylinder stabilizes the flow. A new class of modulated Taylor-Couette flow in which inner cylinder oscillates axially along with constant rotation and stationary outer cylinder has been considered numerically by Hu and Kelly (1995) and Marques and Lopez (1997) and experimentally by Weisberg *et al.* (1997). They observed axial oscillation of the inner cylinder stabilizes the flow. In a recent study Youd *et al.* (2003, 2005) and Youd and Barenghi (2005) studied the stability of Couette flow with fixed outer cylinder and harmonically oscillating inner cylinder around zero mean velocity. They found that, for large amplitude of oscillation both reversing and non-reversing Taylor vortex flow are possible.

Theoretical as well as experimental studies were also carried out for the Rayleigh-Bénard problem in which either the temperature or the gravitational force was externally modulated (Venezian, 1969; Rosenblat and Herbet, 1970; Gresho and Sani, 1970).

The effect of an oscillatory magnetic field on the stability of parallel flow was examined by Drazin (1967). He carried out detailed stability analysis for an inviscid, perfectly conducting fluid.

In all the above discussions, it was found that depending upon the physical parameters, the modulation may destabilize (stabilize) the otherwise stable (unstable) system by parametric modulation.

1.5. Shortcomings in the Literature

The above review briefly described the literature on centrifugal instability, thermal instability and parametric instability. Though the volume of the literature on the hydrodynamic and hydromagnetic stability is very vast, one can readily identify a number of shortcomings in the existing literature which need attention from the researchers.

- Several theoretical, experimental investigations were carried out for modulated Taylor-Couette flow in which modulation is imposed on either the inner, or the outer cylinder. A few theoretical studies were reported on the stability of parallel and convective flows with an oscillating magnetic field but no study has been reported till date on the effect of an oscillating magnetic field on the hydromagnetic Taylor-Couette flow.

- Linear stability analysis of Rayleigh-Bénard problem with boundary temperatures modulated sinusoidally in time are reported in literature with small amplitude of modulation used as an expansion parameter which either consider stress free boundaries or obtains approximate solutions for rigid boundaries. Exact solution can be obtained, if the amplitude of modulation is assumed small and used as an expansion parameter. To lowest order the equations are same as for the unmodulated problem for which the exact solution of Pellew and Southwell (1940) can be used. The equations at higher orders too can be solved in closed form.

- Very substantial amount of theoretical as well as experimental work has been devoted for Taylor vortex flow between two eccentric cylinders, between two conical coaxial cylinders and between two rotating spheres. In all these cases point of instability depends on local Taylor number, which changes from place to place due to change in radius or the gap. No effort has been made till date to study the Taylor vortex flow between coaxial cylinders with step change in radius of one of the cylinders. Instability first sets in the wider gap of the above configuration depending on the local Taylor number.

- Some literature is available for Taylor-Couette flow with two immiscible fluids which are separated by a cylindrical interface but no theoretical study of the stability seems to have been reported with two immiscible fluids between two coaxial cylinders when the interface between the two fluids is planar and perpendicular to the axis of the cylinders.

This thesis aims to study some new configuration and to develop new method to address some of the above issues.

1.6. Organization of Thesis

The present dissertation is organized in six chapters. The first chapter (the present chapter) describes the importance of hydrodynamic and hydromagnetic stability along with parametric instability. A brief review of literature on different types of unstable flow is also presented. Emphasis has been given to the literature directly relevant to the present investigations. Based on this survey some lacunas in the literature have also been pointed out.

In chapter 2, the linear stability of cylindrical Couette flow of an electrically conducting fluid in the presence of an axial magnetic field is examined, where the magnetic field has a small oscillatory component imposed on a steady value. The effect of the field modulation on the threshold of instability is studied for different values of gap width, Chandrasekhar number, magnetic Prandtl number, and oscillation frequency.

In chapter 3, Rayleigh-Bénard problem with the boundary temperatures modulated sinusoidally in time is investigated. The amplitude of modulation is assumed small and is used as an expansion parameter. Exact solution is obtained, even when the boundaries are considered to be rigid. All the possible cases of boundary conditions are studied such as: only the lower boundary temperature is modulated, both boundary temperatures are modulated in phase and the two boundary temperatures are modulated in anti-phase.

In chapter 4, a numerical study of a viscous fluid flow between two coaxial cylinders, where one of the cylinders has a step change in radius is carried out. The inner cylinder undergoes rigid rotation while the outer cylinder is stationary. Computation is restricted to axisymmetric motion since instability in flow between coaxial cylinders is found to first occur in the form of axisymmetric Taylor vortices. In this configuration because of the jump in radius, there are two regions with two different values of the local Taylor number. It would be relevant to study whether the onset of instability in each region is determined by the value of the local Taylor number in that region or there is some influence due to the adjacent region. Moreover, in the absence of bilateral symmetry, it would be relevant to study whether the number of Taylor vortices observed is even or odd.

In chapter 5, linear stability of two axially superposed immiscible fluids between two rotating coaxial cylinders is studied. The fluids are assumed to have equal density but different viscosities. The effect of viscosity ratio of the two fluids on the condition for onset of instability is studied. The critical Taylor number in the less viscous fluid for onset of instability is obtained as a function of the viscosity ratio.

Chapter 6 summarizes the main conclusions of the thesis and points out significant findings from the developed analytical and numerical methodologies.

1.7. Contributions made by the Thesis

This thesis makes the following contributions:

For the Taylor-Couette flow with axially modulated magnetic field, the modulation is shown to be stabilizing for lower value of Chandrasekhar number, then for intermediate range of Chandrasekhar number it is destabilizing and for still higher values of Chandrasekhar number it again has a stabilizing effect.

For the modulated Rayleigh-Benard problem it is shown that if the amplitude of modulation is assumed small and used as an expansion parameter, exact solution for higher order system of equation can be obtained and this gives more accurate results than the approximate solutions reported in literature.

In our numerical study for step cylinder configuration most of the results, whether we obtain even or odd number of vortices in the wide and narrow gap regions, can be understood from the argument that the direction of flow in the Taylor vortices is inward at a stationary end plate but outward at an end plate which has the same rotation rate as the inner cylinder. If the step size is large it functions as another end plate but if the step size is small it has no effect.

In two fluid Taylor-Couette flow, it is found that the variation of the critical Taylor number with viscosity ratio is small when the heights of the fluid columns are large compared to the gap between the cylinders but is significant when the heights are comparable with the gap.