Chapter 1

Introduction

1.1 Introduction

> *'If a man will begin with certainties, he will end with doubts, but if he will be content to begin with doubts, he shall end in certainties.'*
>
> *Francis Bacon (1605)*

Uncertainty is a common phenomenon in real-world decision making problems. When dealing with practical decision problems, it is usually impossible to avoid uncertainty. Hence, in order to achieve a more appropriate outcome of a decision problem, the processing of uncertainties in the information has always been an issue of research since the 18th century. In any problem-solving situation, uncertainty arises due to the presence of different natures of information, such as, incomplete, fragmentary, not-fully-reliable, vague and contradictory. However, there are two basic types of uncertainty present in real-world situations, which are presented in the following:

Probabilistic uncertainty - This type of uncertainty arises due to a lack of information about the future state of the system which may not be known completely. In literature, it is dealt with probability theory.

Linguistic Uncertainty - People often use linguistic terms to express their own perception and subjective knowledge. In a situation where such linguistic terms are interpreted by others, some uncertainty may arise as linguistic terms/words have different or imprecise meaning. One reason behind this is the semantics of languages, which are not unique always. Therefore, one linguistic term can be interpreted in different ways. The standard theory of probability is not well-suited for modeling linguistic uncertainty (Walley and Cooman 2001). Possibility measure (Zadeh 1965) is used to model this type of uncertainty.

Hence, in real-life we have to face the uncertainty which essentially lies in humans expressions. When dealing with practical problems, it is observed that the human attributes like perception, knowledge, experience, attitude, reasoning, thinking etc. cannot be defined at all in the realm of exactness. For example, at the beginning of a consultation session (Dwivedi *et al*. 2006) a doctor asks the patient about his/her condition. When he/she describes the condition, there is inexactness of information viz., source of disease, period of suffering or about symptoms of disease. The doctor, therefore, has to deal with these kinds of incomplete, inexact information. Thus, the proper management of inexact information is necessary for decision making. The research related to the uncertainty which arises due to human's subjective knowledge, would be significant in our highly information-oriented society.

As a matter of fact, several variations of linguistic uncertainty are identified in real-world problems (Carey and Burgman 2008). Fuzziness representing one such variation addresses imprecision or vagueness present in natural language. Vague predicates are often used in normal everyday human communication and fuzzy set theory is a mathematical tool for modeling such vague or imprecise phenomena. In this thesis, we have focused on the uncertainty taken in this context.

The idea of fuzzy sets was borne in 1965 by Professor L. A. Zadeh. Nowadays fuzzy set theory has become widely accepted by scientists and mathematicians, who use it in a wide array of applications, including decision making. Decisions making (Ross 1997) techniques in the deterministic framework have received a lot of attention from researchers over a considerable length of time. With further development of mathematical analysis, different kind of decision making models has become subject of intense research. Decision making models, developed with a view to capture fuzziness, are referred as fuzzy decision making models.

Ever since the theory of fuzzy sets was introduced, there have been large amount of research work accomplished in fuzzy related areas, covering applications and theoretical generalizations. Two years after the emergence of the concept of a fuzzy set, it was generalized by J. Goguen and *L*-fuzzy sets (Goguen 1967) were proposed. Currently there are also some other extensions (e.g., Gorzalczany 1983, Turksen 1986) of fuzzy sets. Out of several higher order fuzzy sets, the concept of intuitionistic fuzzy sets (IFSs), introduced by Atanassov (1986), has been found to be highly useful to deal with vagueness. The major advantage of IFS over fuzzy set may be presented in the following way that, IFS differentiates the degree of membership (belongingness) and the degree of non-membership (non-belongingness) of an element in the set.

Recently there is some terminological debate (Cattaneo and Ciucci 2006) regarding the appropriateness of the name ‘intuitionistic fuzzy sets’. In this regard, the first comment appeared several years before in the paper by Cattaneo and Nistico (1989, p. 183). Later an explicit discussion about this terminological controversy has been published in the paper of Dubois *et al*. (2005). The consequent answers have appeared in the papers of Atanassov (2005), Grzegorzewski and Mrowka (2005). However, in this thesis, we do not get involved in this discussion and merely use the traditional name, i.e., intuitionistic fuzzy sets.

In the next section, the definitions, mathematical operations and the tools required for developing the decision making models in the linguistic framework have been discussed and a brief review of important literature has been presented.

1.2 Preliminaries

1.2.1 Preliminary concepts of fuzzy set theory

The theory of fuzzy sets was first proposed by L. A. Zadeh (1965) in his pioneering paper ‘Fuzzy Sets’ (Zadeh, L.A. (1965). Fuzzy Sets. *Information and Control, 8*, 338-353). Since then a lot of attention has been given on the development of this theory as it provides an excellent mathematical tool to handle vagueness that is inherent in most natural language and in decision-making processes. An in-depth study of fuzzy sets and its applications may be found in the books by Dubois and Prade (1980), Zimmermann (1996), Ross (1997) etc. The concepts and mathematical operations used to model decision making situations with the theory of fuzzy set have been discussed below.

Definition 1.1 **Crisp Set**

A set is a well defined collection of distinct elements or objects. In terms of characteristic function, a crisp set may be defined as follows:

Let X be the universal set and A be any subset of X. Then the characteristic function of A is denoted by χ_A and is defined by the mapping $\chi_A : X \to \{0,1\}$ where

$$\chi_A = \begin{cases} 1, \ x \in A \\ 0, \ x \notin A \end{cases} \qquad \forall x \in X$$

From the above definition it is obvious that for a crisp set, either an element belongs to a set (for which the characteristic function is 1) or does not belong to the set (for which the characteristic function is 0).

Definition 1.2 **Fuzzy Set**

For the case of a fuzzy set, there is no clear sense of belongingness or non-belongingness. This statement becomes clear from the following definition of a fuzzy set. Let X be the universe of discourse under consideration and $\tilde{A}$ be a fuzzy set. Then the fuzzy set $\tilde{A}$ is defined as $\tilde{A} = \{< x, \mu_{\tilde{A}}(x) >: x \in X\}$. Here $\mu_{\tilde{A}}(x)$ is a mapping $\mu_{\tilde{A}}(x) : X \to [0,1]$ and is called the membership function of the fuzzy set $\tilde{A}$. For instance, $\tilde{A}$ may be the set of 'beautiful girls' in a particular class. Since 'beauty' is a subjective concept, such a collection cannot be truly represented by a crisp set. This is where fuzzy set theory comes into play capturing subjective or linguistic evaluation of the situation.

It is to be noted here that the membership function for fuzzy set takes values on the closed interval $[0,1]$. Therefore, there is no sharp boundary between those objects that belong to the set and those that do not. Here $\mu_{\tilde{A}}(x)$ represents the degree of membership or belongingness to the set. So larger the value of $\mu_{\tilde{A}}(x)$, greater is the degree of belongingness and vice versa. The membership function $\mu_{\tilde{A}}(x)$ may be either discrete or continuous. When the universe of discourse is a finite set $X=\{x_1, x_2, ..., x_n\}$, then a fuzzy set $\tilde{A}$ defined on X may be represented as follows:

$$\tilde{A}=\mu_{\tilde{A}}(x_1)/x_1+\mu_{\tilde{A}}(x_2)/x_2+....+\mu_{\tilde{A}}(x_n)/x_n=\sum_{i=1}^{n}\mu_{\tilde{A}}(x_i)/x_i$$

For an infinite universe of discourse X, $\tilde{A}$ may be represented as $A=\int_X \mu_{\tilde{A}}(x)/x$. The family of all fuzzy sets in X is denoted by $FS(X)$.

1.2.2 Fuzzy operators

Some useful fuzzy logic operators (Dubois and Prade 1980, Zimmermann 1996) have been given below. Let $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$ be fuzzy sets defined on *X*, then

Equality:	$\tilde{A}=\tilde{B}$	iff $\mu_{\tilde{A}}(x)=\mu_{\tilde{B}}(x)$	$\forall\ x\in X$
Subset:	$\tilde{A}\subset\tilde{B}$	iff $\mu_{\tilde{A}}(x)\le\mu_{\tilde{B}}(x)$	$\forall\ x\in X$
Union:	$\tilde{C}=\tilde{A}\bigcup\tilde{B}$	iff $\mu_{\tilde{C}}(x)=\max\left\{\mu_{\tilde{A}}(x),\ \mu_{\tilde{B}}(x)\right\}$	$\forall\ x\in X$
Intersection:	$\tilde{D}=\tilde{A}\bigcap\tilde{B}$	iff $\mu_{\tilde{D}}(x)=\min\left\{\mu_{\tilde{A}}(x),\ \mu_{\tilde{B}}(x)\right\}$	$\forall\ x\in X$
Completent:	$\tilde{A}^c$	iff $\mu_{\tilde{A}^c}(x)=1-\mu_{\tilde{A}}(x)$	$\forall\ x\in X$
Normal:		$\mu_{\tilde{A}}(x_0)=1$	if $\exists$ at least one $x_0\in X$
Support of $\tilde{A}$:		$\text{supp}\ \tilde{A}=\{x\in X : \mu_{\tilde{A}}(x)>0\}$	$\forall\ x\in X$
ε-cut:	$[\tilde{A}]_\varepsilon$	$[\tilde{A}]_\varepsilon=\left\{x\in X \mid \mu_{\tilde{A}}(x)\ge\varepsilon\right\}$	$\forall\varepsilon\in[0,1]$

1.2.3 Concept of IFS

An important development of the classical fuzzy sets theory is the theory of IFSs, proposed by Atanassov (1986). In conventional fuzzy set, a membership function assigns to each element of the universe of discourse a number from the unit interval to indicate the degree of belongingness to the set under consideration. The degree of non-belongingness is just automatically the complement to one of the membership degree. However, a human being who expresses the degree of membership of given element in a fuzzy set, very often does not express corresponding degree of non-membership as the complement to one. This reflects a well-known psychological fact that the linguistic negation not always identifies with logical negation (Grzegorzewski 2004). Therefore, Atanassov (1986) suggested a generalization of classical fuzzy set, called IFS. Subsequently Atanassov (1989, 1994, 1999) had also provided important theoretical foundations of IFS. Fundamental knowledge concerning IFSs has been introduced below so as to facilitate further discussion.

Definition 1.3 **Intuitionistic Fuzzy Set (IFS)**

An IFS A in X is given by a set of ordered triples

$$A_{IFS} = \{< x, \mu_A(x), \nu_A(x) >: x \in X\},$$

where $\mu_A, \nu_A : X \to [0,1]$ are functions such that $0 \le \mu_A(x) + \nu_A(x) \le 1 \ \forall \ x \in X$. For each x the numbers $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of non-membership of the element $x \in X$ to $A_{IFS} \subset X$, respectively. Figure 1.1 presents the geometrical interpretation of IFS.

Obviously, each fuzzy set corresponds to the $A_{IFS} = \{< x, \ \mu_A(x), \ 1 - \mu_A(x) >: x \in X\}$, i.e., each fuzzy set is a particular case of the IFS. The family of all IFSs in X is denoted by $IFS(X)$.

For each element $x \in X$ the intuitionistic fuzzy index of x in A_{IFS} is defined as: $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$. The value of $\pi_A(x)$ is called the degree of indeterminacy (or hesitation) of the element $x \in X$ to the IFS *A*. It is observed that $\pi_A(x) \in [0,1]$. If $A \in FS(X)$, then $\pi_A(x) = 0 \;\; \forall\, x \in X$.

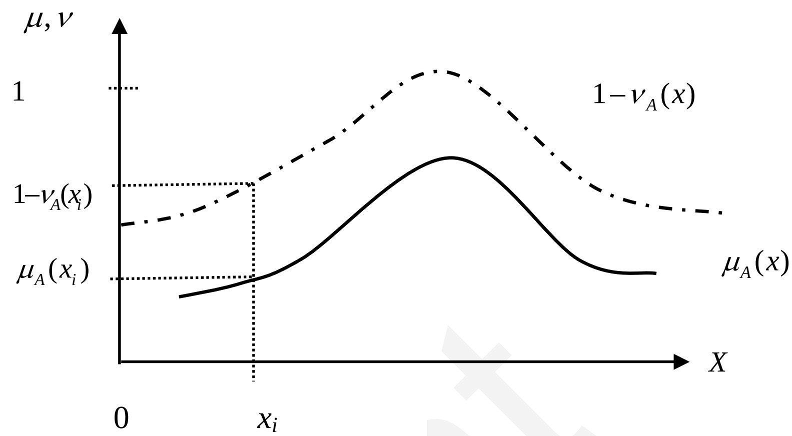

Figure 1.1: Intuitionistic fuzzy set illustration

Definition 1.4 **Operations and Relations over IFSs**

Let A_{IFS}, B_{IFS} be two IFSs defined on the universe of discourse X. For any given element $x \in X$, the following operations are defined as follows:

- **Containment**

$$A_{IFS} \subset B_{IFS} \Leftrightarrow \mu_A(x) \le \mu_B(x) \text{ and } \nu_A(x) \ge \nu_B(x) \quad \forall x \in X$$

- **Equality of two IFSs**

$$A_{IFS} = B_{IFS} \Leftrightarrow \mu_A(x) = \mu_B(x) \text{ and } \nu_A(x) = \nu_B(x) \quad \forall x \in X$$

- **Complement of IFS**

$$A_{IFS}{}^{c} = \{< x, \nu_A(x), \mu_A(x) >: x \in X\}$$

1.3 Linguistic variable and fuzzy number

A linguistic variable (Zadeh 1975) may be regarded either as a variable whose value is a fuzzy number or as a variable whose values are defined in linguistic terms. As for example, while answering the question ‘How is the weather today?’ one might observe ‘It is warm’. The linguistic variables like ‘warm’, so common in everyday speech, convey information about our environment. But this information is unquantifiable as it is stated in linguistic terms. Specifically, to model decision making problems experts often use words or predicates in natural languages while expressing the necessary information. Since in general, real-world instances possess vagueness, linguistic variables are used to represent experts' perception, expression, knowledge etc. In this scenario, fuzzy mathematics becomes a natural choice since it can define the linguistic information in a more logical and meaningful fashion. The mathematical formalism of linguistic variable may be presented in the following way:

Definition 1.5 **Linguistic Variable**

A linguistic variable is characterized by a quintuple $(x, \Im(x), X, G, M)$, in which x is the name of the linguistic variable; $\Im(x)$ is the term set of x, i.e., the set of names of linguistic values of x with each value being a fuzzy number defined on X; G is a syntactic rule for generating the names of values of x; and M is a semantic rule for associating with each value its meaning. The family of all fuzzy (sub) sets in X is denoted by $\Im(X)$.

1.4 Basic definition and notation of fuzzy numbers

Definition 1.6 **Fuzzy Number**

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set defined on the universe of discourse $\mathbb{R}$ (the set of all real numbers) with a piecewise continuous membership function and bounded support.

Definition 1.7 **L-R Representation of Fuzzy Number**

A fuzzy number $\tilde{A}$ is said to be an L-R type fuzzy number (Dubois and Prade 1980) iff

$$\mu_{\tilde{A}}(x) = \begin{cases} \mathrm{L}((m_1 - x)/\beta), & \text{for} \quad m_1 - \beta \le x \le m_1 \\ 1 & \text{for} \quad m_1 \le x \le m_2 \\ \mathrm{R}((x - m_2)/\gamma), & \text{for} \quad m_2 \le x \le m_2 + \gamma \end{cases}$$

L and R are called the left and right reference functions, m_1, m_2 are the left point, right point and β, γ the left and right spreads, respectively.

These reference functions satisfy the following conditions:

(*i*) $\mathrm{L}(x) = \mathrm{L}(-x)$

(*ii*) $\mathrm{L}(0) = 1$

(*iii*) $\mathrm{L}(\cdot)$ is non-increasing on $[0, \infty)$

The left and right shape functions may be both linear and non-linear. For example, $\mathrm{L}(x) = 1, x \in [-1,1]$, $\mathrm{L}(x) = e^{-|x|^p}, p \geq 0$, $\mathrm{L}(x) = \max(0, 1 - |x|^p), p \geq 0$ are instances of the left reference function. The right reference function may be defined similarly. The fuzzy number $\tilde{A}$ is denoted as $\tilde{A} = (m_1, m_2; \beta, \gamma)_{\mathrm{LR}}$.

Definition 1.8 **Generalized Fuzzy Number (GFN)**

A GFN $\tilde{A} = (m_1, m_2; \beta, \gamma; w)$ (Chen 1985, 1999) is a fuzzy subset on the real line $\mathbb{R}$, where $0 < w \leq 1$, and m_1, m_2, β and γ are real numbers. The membership function $\mu_{\tilde{A}}$ of $\tilde{A}$ satisfies the following conditions:

(*i*) $\mu_{\tilde{A}}$ is a continuous mapping from $\mathbb{R}$ to the closed interval $[0, w]$, $0 < w \leq 1$;

(*ii*) $\mu_{\tilde{A}}(x) = 0$, where $-\infty < x \leq m_1 - \beta$;

(*iii*) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[m_1 - \beta, m_1]$;

(*iv*) $\mu_{\tilde{A}}(x) = w$, where $m_1 \leq x \leq m_2$;

(*v*) $\mu_{\tilde{A}}(x)$ is strictly decreasing on $[m_2, m_2 + \gamma]$;

(*vi*) $\mu_{\tilde{A}}(x) = 0$, where $m_2 + \gamma \leq x < \infty$.

- If $\mu_{\tilde{A}}(x)$ is a linear function of *x*, $\tilde{A}$ is called a generalized trapezoidal fuzzy number.
- If $\mu_{\tilde{A}}(x)$ is a linear function of x and also $m_1 = m_2 = m$ then $\tilde{A}$ is called a generalized triangular fuzzy number (GTFN).
- If $w = 1$, then the GFN $\tilde{A}$ is called a normalized fuzzy number and denoted as $\tilde{A} = (m_1, m_2; \beta, \gamma)$. If $\beta = 0$ and $\gamma = 0$, then $\tilde{A}$ is called a crisp interval.
- If $\beta = 0$, $\gamma = 0$, $m_1 = m_2$ and $w = 1$, then $\tilde{A}$ is called a real number.

If a GFN $\tilde{A}$ denotes an expert's opinion, then the value w denotes the corresponding degree of confidence of the expert. Without any loss of generality throughout the thesis, to represent the linguistic assessments provided by the experts in a flexible way, we have considered GTFN of the form $\tilde{A} = (m;\beta,\gamma;w)$ with m, β, γ as positive real numbers and $0 < w \le 1$.

Definition 1.9 **ε-cut Representation**

The ε-cut representation of a GFN $\tilde{A} = (m_1, m_2;\beta,\gamma;w)$ is denoted by $[A^L(\varepsilon), A^R(\varepsilon)]$ and is defined by

$$A^L(\varepsilon) = \begin{cases} \inf\,\{x \mid \mu_{\tilde{A}}(x) \ge \varepsilon\} & \text{if } 0 < \varepsilon \le w \\ \inf\,\{x \mid x \in [m_1 - \beta, m_2 + \gamma]\} & \text{if } \varepsilon = 0 \end{cases} \text{ and}$$

$$A^R(\varepsilon) = \begin{cases} \sup\,\{x \mid \mu_{\tilde{A}}(x) \ge \varepsilon\} & \text{if } 0 < \varepsilon \le w \\ \sup\,\{x \mid x \in [m_1 - \beta, m_2 + \gamma]\} & \text{if } \varepsilon = 0 \end{cases}.$$

Indeed, the ε-cut of a fuzzy number is an interval number, i.e., $[\tilde{A}]_\varepsilon = [A^L(\varepsilon), A^R(\varepsilon)]$.

In order to manipulate algebraically with intervals numbers, derived from the fuzzy numbers, the following interval arithmetic operations have been used.

Definition 1.10 **Interval Arithmetic**

Let us consider two interval numbers [a, b] and [c, d] where $a \le b$ and $c \le d$. Then the operations are given as follows:

(i) Addition: $[a,b]+[c,d] = [a+c, b+d]$
(ii) Subtraction: $[a,b]-[c,d] = [a-d, b-c]$
(iii) Multiplication: $[a,b]\times[c,d] = [\min\{ac, ad, bc, bd\}, \max\{ac, ad, bc, bd\}]$
(iv) Division: $[a,b] \div [c,d] = [a,b]\times[1/d, 1/c]$

Definition 1.11 **Ranking of GFNs**

In comparing GFNs, researchers have introduced various ranking methods. One of such methods that have been employed in the thesis is a centroid-based ranking method. The method has been discussed briefly as follows:

The centroid point $(\bar{x}(\tilde{A}), \bar{y}(\tilde{A}))$ for a generalized trapezoidal fuzzy number $\tilde{A} = (m_1, m_2; \beta, \gamma; w)$ is defined (Wang *et al*. 2006) as follows:

$$\bar{x}(\tilde{A}) = \frac{1}{3}.\left[2m_1 + 2m_2 - \beta + \gamma - \frac{m_2(m_2+\gamma) - m_1(m_1-\beta)}{(2m_2+\gamma)-(2m_1-\beta)}\right] \text{ and}$$

$$\bar{y}(\tilde{A}) = w\frac{1}{3}.\left[1 + \frac{m_2 - m_1}{(2m_2+\gamma)-(2m_1-\beta)}\right]$$

In particular, for a GTFN of the form $\tilde{A} = (m; \beta, \gamma; w)$, its centroid is determined by:

$$\bar{x}(\tilde{A}) = \frac{1}{3}.\left[3m - \beta + \gamma\right] \text{and } \bar{y}(\tilde{A}) = \frac{1}{3}w$$

For any two different GFNs $\tilde{A}_1$ and $\tilde{A}_2$, the ranking (Wang and Lee 2008) may be done in the following way:

*(a)*If $\bar{x}(\tilde{A}_1) > \bar{x}(\tilde{A}_2)$ then $\tilde{A}_1 > \tilde{A}_2$

(b) If $\bar{x}(\tilde{A}_1) < \bar{x}(\tilde{A}_2)$ then $\tilde{A}_1 < \tilde{A}_2$

(c) If $\bar{x}(\tilde{A}_1) = \bar{x}(\tilde{A}_2)$ then,

if $\bar{y}(\tilde{A}_1) > \bar{y}(\tilde{A}_2)$ then $\tilde{A}_1 > \tilde{A}_2$;

else if $\bar{y}(\tilde{A}_1) < \bar{y}(\tilde{A}_2)$ then $\tilde{A}_1 < \tilde{A}_2$;

else $\bar{y}(\tilde{A}_1) = \bar{y}(\tilde{A}_2)$ then $\tilde{A}_1 = \tilde{A}_2$.

Basically, in the above ranking process $\tilde{A}_1$ and $\tilde{A}_2$ are ranked based on their $\bar{x}$'s values if they are different. In the case that they are equal, we further compare their $\bar{y}$'s values to rank them.

1.5 Fuzzy reasoning for decision modeling

The study of decision problems has a long history and in the last few decades has been one of the major research fields in decision sciences. In real-world, people have to make lots of decisions during their life in the presence of multiple, usually conflicting criteria. Multiple criteria decision making (MCDM) addresses the general class of problems that involve multiple attributes, objectives and goals. According to many authors (e.g., Zimmermann 1996) MCDM may be broadly classified into two categories: multiple attribute decision making (MADM) and multiple objective decision making (MODM). The main difference between MODM and MADM (Lu *et al*. 2007) is that the former concentrates on continuous decision spaces with several objective functions and the latter focuses on problems with discrete decision spaces.

The main characteristic of MODM problem is that the decision maker wants to achieve multiple objectives while these multiple objectives are conflict with each other. But the main difficulty is that usually there is no optimal solution for all the objectives. Therefore, the decision maker conducts some tradeoff analysis among different objectives to determine an acceptable solution.

MADM method, on the other hand, concentrates on problems with a limited number of predetermined alternatives. MADM approaches begin with the task of finding the relevant attributes and alternatives. In MADM techniques, attributes are characteristics, qualities, factors, performance indices etc. Subsequently depending on multiple attributes decision maker selects the best among different alternatives.

However, the first step towards a suitable decision for any given decision making problem is to develop a mathematical framework representing it. In general, a real-world decision making process consists of four stages, which may be represented as information gathering, mathematical modeling, simulation and decision/action followed in a sequence. In this process, due to lack or abundance of information, subjective estimation or vagueness and incomplete knowledge about the complex systems, linguistic variables are used to represent expert's perception, expression, knowledge etc. Therefore, the primary task may be regarded as the capturing of these linguistic expressions within a mathematical framework for the sake of mathematical model building. Fuzzy set theory, due to its association with approximate reasoning (Chakraborty and Chakraborty 2004), is capable of quantifying these linguistic expressions and, thus, provides a systematic approach to deal with decision situations in which the information cannot be assessed exactly in a quantitative form (Delgado *et al*. 1992, Hsu and Chen 1997). In view of this, fuzzy expert systems (Jin 2000, Yu and Bien 1994) are developed based on the fuzzy knowledgebase (KB). The fuzzy knowledge-based system helps us to include the verbal and mathematical description of the real complicated systems. Moreover, it also facilitates in developing the techniques for the solution of real-life decision problems which inherently possess vagueness.

Fuzzy knowledge-based computational models are potentially very useful for complex problems in different decision making areas. One such area, i.e., MCDM has been discussed earlier. Another area is fuzzy statistical decision making. The theory of fuzzy statistics has widened the scope of statistics enabling us to deal with more general sources of uncertainty, such as, vagueness and imprecision. In recent years, many of the fuzzy ideas and techniques have been translated into the statistical language, yielding new methodological proposals, especially in the areas of regression. Statistical regression is problematic if the data set is too small or if the data is somewhat corrupted or if there is vagueness in the relationship between the independent and dependent variables (Chakraborty and Chakraborty 2008). These are the very situations fuzzy regression was meant to address. In spite of the impact of the growing literature, there is relevant area for further developments in several directions, including methodology and applications of the fuzzy approach to regression analysis. In order to make a connection between fuzzy set theory and statistical regression methods, an attempt has been made in the thesis for regression modeling with linguistic variables.

1.6 Fuzzy inference methodology

The KB of a fuzzy expert system is a set of fuzzy decision rules instead of crisp rules to represent expert's knowledge, experience, perception, reasoning and thinking (Chakraborty and Chakraborty 2007a). A fuzzy decision rule is the basic unit for capturing subjective information in many fuzzy systems. A fuzzy rule has two components: an if-part (also referred to as the antecedent) and a then-part (also referred to as the consequent):

IF premise (antecedent), THEN conclusion (consequent).

Fuzzy if-then rules have been applied to many disciplines, such as, control systems, decision making, medical imaging, pattern recognition. Basically, a decision making in fuzzy rule base, i.e., fuzzy rule based inference consists of the three basic steps and an additional optional step (Yen and Langari 2005) presented as follows:

- *Fuzzy matching*: Calculate the degree to which the input data match the condition of fuzzy rules.
- *Inference*: Calculate each of the fuzzy rule's conclusion based on the matching degree.
- *Combination*: Combine the conclusion inferred by all fuzzy rules into a final conclusion.
- *Defuzzification* (optional): For applications that need a crisp output, an additional step is suggested to convert a fuzzy conclusion into a crisp output.

Mathematicians, specialists on fuzzy logic, developed many kinds of fuzzy inference systems. There are three widely used fuzzy inference system: Mamdani (Mamdani and Assilian 1975), Tsukamoto (Tsukamoto 1979) and Takagi-Sugeno (Takagi and Sugeno 1985). The basic difference between various models lies in the representation of the consequents of their fuzzy rules. A brief description of the three well-known fuzzy inference systems has been given as follows:

1.6.1 Mamdani inference scheme

E.H. Mamdani and S. Assilian (Mamdani and Assilian 1975) proposed the first type of fuzzy rule based Systems. The rules in a Mamdani fuzzy system are specified linguistically both for antecedents and consequents. Here Mamdani's reasoning method has been briefly described. Consider the following system of *p* fuzzy rules:

$$\begin{array}{ll}
\Re_j: \text{If } x_1 \text{ is } \tilde{A}_{j1} \text{ and } x_2 \text{ is } \tilde{A}_{j2} \text{ and ....and } x_n \text{ is } \tilde{A}_{jn} \text{ then } z \text{ is } \tilde{C}_j. \\
\text{Input: } x_1 \text{ is } y_1 \text{ and } x_2 \text{ is } y_2 \text{ and ....and } x_n \text{ is } y_n \\
\hline
\text{Output:} \qquad\qquad\qquad\qquad\qquad\qquad\qquad z \text{ is } \tilde{C}.
\end{array}$$

Here $\tilde{A}_{jk} \in \Im(X_k)$ is a value of the linguistic variable x_k defined in the universe of discourse $X_k \subset \mathbb{R}$ and $\tilde{C}_j \in \Im(Z)$ is a value of the linguistic variable z defined in the universe $Z \subset \mathbb{R}$, where $j = 1, 2, ..., p$ and $k = 1, 2, ..., n$. The crisp input vector is $y = \{y_1, y_2, ..., y_n\}$.

Then the overall system output fuzzy set $\tilde{C}$ on Z for the given fuzzy rule base is computed by $\mu_{\tilde{C}}(z) = \overset{p}{\underset{j=1}{\vee}}[l_j \wedge \mu_{\tilde{C}_j}(z)]$, where l_j is the degree to which the input matches the jth rule $\Re_j$ and is computed by $l_j = \overset{n}{\underset{k=1}{\wedge}} \mu_{\tilde{A}_{jk}}(y_k)$ (Here $\wedge$ is the *minimum* operator and $\vee$ is the *maximum* operator).

Here the output $\tilde{C}$ is a fuzzy set. Therefore, finally to obtain a deterministic action, this fuzzy output can be defuzzified using any defuzzification strategy.

1.6.2 Tsukamoto's inference scheme

Tsukamoto's (Tsukamoto 1979) inference scheme is characterized by the following:

- In this model the consequent of each fuzzy if-then-rule is represented by a fuzzy set with a strictly monotone membership function.

A brief description of Tsukamoto's fuzzy reasoning method has been given below. For this purpose, consider the following fuzzy inference system:

$\Re_j$: If x_1 is $\tilde{A}_{j1}$ and x_2 is $\tilde{A}_{j2}$ and ….and x_n is $\tilde{A}_{jn}$ then z is $\tilde{C}_j$.

Input: x_1 is y_1 and x_2 is y_2 and ….and x_n is y_n

Output: z is z_{TS}.

where $\tilde{A}_{jk} \in \Im(X_k)$ is a value of the linguistic variable x_k defined in the universe of discourse $X_k \subset \mathbb{R}$, and $\tilde{C}_j \in \Im(Z)$ is a value of the linguistic variable z defined in the universe $Z \subset \mathbb{R}$ for $j = 1, 2, ..., p$ and $k = 1, 2, ..., n$. It is assumed that Z is bounded. In

Tuskamoto's (1979) fuzzy reason scheme it is supposed that each $\tilde{C}_j$ has strictly monotonic membership functions on Z .

The procedure for obtaining the crisp output z_{TS}, from the crisp input vector $y = \{y_1, y_2, ..., y_n\}$ and fuzzy rule base $\{\Re_1, \Re_2, .., \Re_p\}$ are described as follows:

- The degree to which input matches the *j*th rule $\Re_j$ is computed by $l_j = t(\mu_{\tilde{A}_{j1}}(y_1), \mu_{\tilde{A}_{j2}}(y_2), ..., \mu_{\tilde{A}_{jn}}(y_n))$, for $j = 1, 2, ..., p$, where *t* defines a general class of intersection operators of fuzzy sets. In Tsukamoto's inference scheme usually the product or minimum operator is used.
- In this mode of reasoning, the individual crisp control actions z_j is derived from the relation $l_j = \mu_{\tilde{C}_j}(z_j)$, i.e., $z_j = \mu_{\tilde{C}_j}^{-1}(l_j)$ (The inverse of $\mu_{\tilde{C}_j}$ is well-defined because of its strict monotonicity).
- The overall system output is defined as the weighted average of the individual outputs, where associated weights are the firing levels. Therefore, the overall crisp control action is computed by $z_{TS} = \dfrac{l_1 z_1 + ... + l_p z_p}{l_1 + ... + l_p}$.

1.6.3 Takagi and Sugeno fuzzy reasoning scheme

Takagi and Sugeno (1985) introduced a fuzzy reasoning scheme. In this fuzzy reasoning method, a fuzzy rule is of the following form:

'If x is $\tilde{A}_1$ and y is $\tilde{A}_2$ then $z = f(x, y)$'

where $\tilde{A}_1$ and $\tilde{A}_2$ are fuzzy sets in the antecedent, while $z = f(x, y)$ is a crisp function in the consequent, $f(.,.)$ is very often a linear function with respect to *x* and *y*. Here we briefly discuss the Takagi-Sugeno model. Let us consider the following architecture:

$\Re_j$: If x_1 is $\tilde{A}_{j1}$ and x_2 is $\tilde{A}_{j2}$ and …and x_n is $\tilde{A}_{jn}$ then $z = a_{j1}x_1 + a_{j2}x_2 + ... + a_{jn}x_n + b_j$

Input: x_1 is y_1 and x_2 is y_2 and ….and x_n is y_n

Output: z is z_{TgS}

where $\tilde{A}_{jk} \in \Im(X_k)$ is a value of the linguistic variable x_k defined in the universe of discourse $X_k \subset \mathbb{R}$, a_{jk} and b_j are real numbers for $j = 1, 2, ..., p$ and $k = 1, 2, ..., n$. The procedure for obtaining the crisp output z_{TgS}, from the crisp input vector $y = \{y_1, y_2, ..., y_n\}$ and fuzzy rule base $\{\Re_1, \Re_2, .., \Re_p\}$ are described as follows:

- The degree to which input matches the *j*th rule $\Re_j$ is typically computed using the relation $l_j = \prod_{k=1}^{n}(\mu_{\tilde{A}_{jk}}(y_k))$, for $j = 1, 2, ..., p$.
- Then the individual rule outputs derived from the relationship $z_j(y) = \sum_{k=1}^{n} a_{jk}y_k + b_j$.
- Finally, the crisp control action is represented as: $z_{TgS} = \dfrac{l_1 z_1 + ... + l_p z_p}{l_1 + ... + l_p}$.

The main difference between Mamdani fuzzy reasoning schemes and Takagi-Sugeno fuzzy reasoning methods lies in the consequent of the fuzzy rules: the former uses the fuzzy sets where the latter employs the (linear) functions of input variables. Tsukamoto's fuzzy reasoning schemes use fuzzy sets with monotonic membership functions for the consequent of the rule base.

However, for making decision from fuzzy rule based system, depending on the nature of the consequent of fuzzy if-then rules, two traditional well known fuzzy inference mechanisms that have been employed in the thesis are: Takagi-Sugeno fuzzy inference scheme and Tsukamoto fuzzy inference scheme.

1.7 Motivation and objective of the work done

The goal of the thesis is to explore the applicability of fuzzy framework in developing realistic decision making models. In view of this objective, this work focuses on the development of fuzzy expert systems in a single and group decision making environment. The thesis not only focuses on theoretical aspects of various decision problems, but also develops tools and techniques for information processing and subsequent modeling leading to an efficient decision support system in linguistic framework. Moreover, in order to demonstrate the developed methodologies different types of real-world applications have also been considered. The genesis of the works has been presented below. The means of realizing the motive behind the work done has also been outlined.

1.7.1 An overview of GFNs: theory and applications

In order to handle uncertain information with more flexibility, Chen (1985) introduced the concepts of GFNs following the study of normalized fuzzy numbers. In this respect, Chen (1985, 1999) had also formulated different arithmetic operations on GFNs as traditional fuzzy arithmetic operations can only deal with normalized fuzzy numbers. Later a few researches have been done on GFN theory and its applications (e.g., Chen and Chen 2003, Wei and Chen 2009). Some other researchers also developed methodologies for ranking of GFNs (e.g., Chen and Chen 2007, Chen and Chen 2009, Chen and Wang 2009, Lee and Chen 2008).

It has been observed from the aforementioned literature review that in most of the papers on GFNs, to deal with the arithmetical operations, Chen's arithmetic operators are used. Eventually it has been realized that Chen's arithmetic operators make some approximation, therefore, the exact result may not be found. This is the motivation of Chapter 2 to develop the arithmetic operators on GFNs in a way so that the drawbacks of existing operators are overcome.

Chapter 3 studies the distance measure and similarity measure of GFNs. Distance measures have become important due to its significant applications in diverse fields like decision making, remote sensing, data mining, pattern recognition, multivariate data analysis etc. Several distance measures for precise numbers are well established in the literature. Many researches have also been done to construct the distance measure between fuzzy sets (e.g., Kacprzyk 1997). Recently, some researchers have focused their attention to compute the distances between fuzzy numbers, such as, Cheng (1998) introduced a distance index based on the centroid points. Diamond (1988) defined a measure for fuzzy numbers in the Euclidean space. Yang and Ko (1997) modified Diamond's proposed measure. A fuzzy distance measure for gaussian type fuzzy numbers was defined by Xu and Li (2001).Tran and Duckstein (2002) introduced the distance concept based on the interval numbers where the fuzzy number is transformed into an interval number on the basis of the ε-cut. However, in the literature it has been observed that the distance methods basically compute crisp distance measure for fuzzy numbers. In this regard, Voxman (1998), Chakraborty and Chakraborty (2006) proposed fuzzy distance measures that can compute the distance between two normalized fuzzy numbers.

However, eventually some shortcomings of existing fuzzy distance measures (Chakraborty and Chakraborty 2006, Voxman 1998) have been observed. In view of this, a new fuzzy distance measure that can calculate the distance measure between two GFNs has been developed in Chapter 3.

Moreover, in case of fuzzy reasoning, the question of selecting the suitable fuzzy similarity measure is also very essential. Similarity measure between two fuzzy numbers is related to their commonality, in theories of the recognition, identification and categorization of objects, where a common assumption is that the greater the commonality between a pair of objects, more similar they are. Different methods were presented to calculate the degree of similarity between normalized fuzzy numbers by several authors (e.g., Chakraborty and Chakraborty 2004, Chen 1996, Hsieh and Chen 1999, Lee 1999). Some researchers also have paid considerable attentions to the research on similarity measures between GFNs, such as, Chen and Chen (2003) proposed a similarity measure for GFNs based on center-of-gravity (COG) points. In 2004, Yong *et al*. proposed a new similarity measure of GFNs based on the radius of gyration points. Sridevi and Nadarajan (2009) proposed a similarity measure for GFNs based on fuzzy difference of distance of points of fuzzy numbers. Combining the concepts of geometric distance, the perimeter and the height of GFNs, Wei and Chen (2009) proposed a new similarity measure between GFNs. Subsequently Xu *et al*. (2010) analyzed the similarity measure of Wei and Chen (2009) and proposed another new similarity measure for GFNs.

It has been observed from the literature review presented above that all the existing similarity measures give crisp values for the similarities between two fuzzy numbers. However, often in a decision making environment it is said that two experts have 'much similarity' in their opinions, which is fuzzy in nature. In this regard, a new concept of fuzzy similarity measure of GFNs has also been introduced in Chapter 3.

1.7.2 MADM technique under fuzzy environment

In this section, MADM situations under fuzzy environment have been considered.

Over the last few decades fuzzy MADM techniques have attracted researches due to its applicability for solving real-world decision making problems. For instance, Biswas (1995) described a computer based fuzzy evaluation method for students' grading. A problem of job evaluation under fuzzy environment (Gupta and Chakraborty 1998) was dealt using fuzzy goal programming technique. Fuzzy MADM models and methods with incomplete preference information were studied by Li (1999). A man-machine interactive algorithm (Chakraborty 2001, 2002) was designed to represent the human perception into a fuzzy mathematical structure by acquisition of linguistic information. In 2004, Li and Yang developed a linear programming technique to solve MADM problem.

Another very well known MADM approach, which was introduced by Hwang and Yoon (1981), is Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). Later researchers have extended TOPSIS technique in fuzzy environment and an impressive variety of fuzzy TOPSIS algorithms and applications are developed in recent years. In 2002, Tsaur's *et al*. evaluated the service quality of airlines by applying TOPSIS in ranking methodology. Chen and Tzeng (2004) combined grey relation and TOPSIS for selecting an expatriate host country. Abo-Sinna and Amer (2005), Abo-Sinna *et al*. (2008) extended the TOPSIS for large scale multi-objective non-linear programming problems. Chen *et al*. (2006) developed a fuzzy decision making approach to deal with the supplier selection problem in supply chain system using the concept of TOPSIS.

Wang and Elhag (2006) developed a non-linear programming solution procedure using fuzzy TOPSIS method, which is based on alpha level concept. Jahanshahloo *et al*. (2006) extended TOPSIS method for decision making problems with fuzzy data**.** A fuzzy hierarchical TOPSIS model for the multi-criteria evaluation of the industrial robotic systems was proposed by Kahraman *et al*. (2007). Wang and Chang (2007) developed an evaluation approach based on fuzzy TOPSIS to choose optimal initial training aircraft. Wang and Lee (2007) generalized TOPSIS in a fuzzy environment by proposing two operators *Up* and *Lo* to find the ideal solution and negative ideal solution. Kuo *et al*. (2007) presented a fuzzy multi-criteria decision method based on the concepts of ideal and anti-ideal points. Li (2007) developed compromise ratio (CR) methodology. Benitez *el al*. (2007) applied a fuzzy TOPSIS methodology for evaluating the service quality of three hotels of an important corporation in Gran Canaria Island. Nezhad and Damghani (2010) assessed the performance of traffic police centers using a new TOPSIS method in fuzzy environment. Extensions of the TOPSIS for group decision making (GDM) under fuzzy environment by Chen (2000) could be assumed as one of the prior work in this field.

Now in practice, any group decision (Kim and Ahn 1999, Kim *et al*. 1999, Xu 2000) completely depends on the available linguistic information. Ideally, it is expected that the linguistic assessments provided by the experts for the same criteria should be close to each other to some extent (Chakraborty and Chakraborty 2007b). But in reality the homogeneity among the responses of the experts cannot always be maintained due to diversity among the subjective human factors like knowledge, skills, experience, personality etc. In view of this, there is a need to automate a decision process to aggregate the fuzzy information given by individual expert into a group consensus opinion. A brief review of literature regarding the aggregation technique under fuzzy environment has been given next.

Nurmi (1981) and Tanino (1984) introduced fuzzy preference relation to compute the strength of group fuzzy preference relation. Kacprzyk and Fedrizzi (1988), Kacprzyk *et al*. (1992) presented 'soft' degrees of consensus and employed the fuzzy linguistic quantifiers to represent a fuzzy majority. Hsu and Chen (1996) proposed a similarity aggregation method. Bordogna *et al*. (1997) proposed a model for GDM in a linguistic context to evaluate a consensus judgment and a consensus degree on each alternative. Herrera *et al*. (1996, 1997) developed a consensus model in GDM under linguistic assessments. Herrera-Viedma *et al*. (2005) introduced a model of consensus support system to assist the experts in all phases of the consensus reaching process of GDM problems with multigranular linguistic preference relations. Xu (2005) studied the GDM problem with linguistic preference relations. Ben-Arieh and Chen (2006) presented a procedure for handling an autocratic GDM process under linguistic assessments. Ekel *et al*. (2009) addressed the use of a consensus scheme for achieving a consistent aggregation of the individual experts' opinions. Cabrerizo *et al*. (2010) presented a model of consensus for GDM problems with incomplete unbalanced fuzzy linguistic information. Khorshid (2010) proposed a model for developing a soft consensus between decision makers under a fuzzy environment.

Thus, as evident from the literature review presented above, fuzzy MADM models have been focused on by various researchers in the recent past.

However, our view point is something different from the previous works. In any real decision making situation when an expert gives his response to a particular alternative, his/her confidence level with respect to such queries is important. In this respect, a GFN may be considered as a more suitable and general mathematical representation of linguistic expressions of the experts since it considers the degrees of confidence of experts' opinions. For example, two experts give their responses to a particular query: first expert says 'I have *full confidence* in his personality being suitable for the job under consideration'. Another says that 'I am *somewhat confident* about his personality being suitable for the job under consideration'. It is to be noted here that both the experts opine that the candidate's personality is suitable for the job but their opinions vary in their individual confidence levels. So there may be such situations where evaluation of an alternative may be dependent on the confidence level of an expert. In a brief review of the literature, it has been observed that the problem of MADM considering the confidence level of an expert has not yet been attended so far and this is the aspect that has motivated the work of Chapter 4 and Chapter 5. In particular, with due consideration of the degrees of confidence of experts' opinions, Chapter 4 and Chapter 5 provide techniques to solve fuzzy MADM problems in single and group decision making environments, respectively.

1.7.3 Theoretical developments and applications of IFSs

Chapter 6 and Chapter 7 focus on IFSs, which is regarded as another strong framework for handling uncertainty of real-world. In 1986, Atanassov introduced the concept of IFS. Subsequently Gau and Buehrer (1993) introduced the concept of vague set, which is another generalization of fuzzy set. Bustince and Burillo (1996) pointed out that the notion of vague set is the same as that of IFS.

Out of several higher order fuzzy sets, the concepts of IFSs have been found to be highly useful to deal with vagueness. Over the last few decades the IFS theory has been investigated by many authors (Bustince *et al*. 2000, Mondal and Samanta 2001, 2002, Szmidt and Kacprzyk 2001 etc.) and applied in different fields, like decision making (e.g., Atanassov *et al*. 2005, Chen and Tan 1994, Hong and Choi 2000, Li 2005, Lin *et al*. 2007, Liu and Wang 2007, Szmidt and Kacprzyk 2002, Xu and Yager 2008 and 2009, Wei 2010, Ye 2010), logic programming (e.g., Atanassov and Georgiev 1993), medical diagnosis (e.g., De *et al*. 2001) etc.

Since Atanassov originated the idea of IFSs, different similarity measures between IFSs have been proposed in the literature (e.g., Chen 1995, 1997, Hong and Kim 1999, Zhang and Fu 2006). Some researchers proposed similarity measures for IFSs and applied them in pattern recognition problems (e.g., Hung and Yang 2007, Li and Cheng 2002, Liang and Shi 2003, Liu 2005, Mitchell 2003). A set of similarity measures for IFSs were proposed by Xu (2007d) and also successfully applied in decision making problem.

After analyzing the existing similarity measures it has been observed that, for some cases, they fail to calculate the measures of similarities correctly. In order to overcome these problems, a new similarity measure for IFSs has been proposed in Chapter 6. Subsequently a methodology has also been developed for estimating priority-based weights of the alternatives using the proposed similarity measure under intuitionistic fuzzy environment.

As important contents in fuzzy mathematics, distance measures between IFSs have also attracted many researchers. Several researchers focused on computing the distance between IFSs. Atanassov (1999), Szmidt and Kacprzyk (2000) suggested some distances measures for IFSs, which are the generalizations of the well-known normalized Hamming distance and normalized Euclidean distance. Subsequently Grzegorzewski (2004) proposed another group of distance measures for IFSs based on Hausdorff metric. Wang and Xin (2005) had shown that the distance measure proposed by Szmidt and Kacprzyk (2000) is not reasonable for some cases and, thereafter to overcome its drawback they provided two new distance measures for IFSs. Huang *et al*. (2005) developed a group of distance measures to unify the distances proposed by Atanassov (1999) and Grzegorzewski (2004). After that they proposed a new group of distance measures for IFSs**.** Hung and Yang (2007) defined another new fuzzy distance measure for calculating the distance for IFSs based on L_p metric. Grzegorzewski (2003) investigated two families of metrics in space of intuitionistic fuzzy numbers (IFNs). However, the distance measures proposed by Grzegorzewski (2003) compute crisp distance measures for IFNs.

However, the well-known fact that needs to be remembered here is that if the uncertainty is inherent within the numbers, this uncertainty should be intrinsically connected with their distance value. This is the reason why fuzzy distance measure for measuring the distance measure between two fuzzy numbers came into the field. For the same reason it is not reasonable to define crisp distance between IFNs. With this view point, in Chapter 7 a new methodology to measure the distance for IFNs has been proposed, which will provide an IFN as the outcome.

In order to introduce some of the key issues in decision making techniques in imprecise environment we have worked on a number of MADM models to help decision makers solving complex decision problems in a more systematic, consistent and productive way.

Another fundamental MCDM model that has attracted the attention of researchers for a long time is MODM, which has been highlighted in Chapter 8.

1.7.4 MODM under fuzzy framework

In the conventional optimization problems, the coefficients are all assumed as real numbers. When the coefficients are taken as fuzzy numbers, the optimization problems with fuzzy coefficients should be solved by invoking the efficient methodology in the field of fuzzy optimization. Bellman and Zadeh (1970) developed fuzzy optimization problems by providing the aggregation operators to combine the fuzzy goals and fuzzy decision space. Zimmermann (1978) first used the fuzzified constraint and objective functions to solve the multi-objective linear programming problems. Chanas (1989) used the parametric programming technique to solve the fuzzy multi-objective linear programming problems. After this motivation and inspiration there come out a lot of literature dealing with the fuzzy optimization problems (e.g., Ali 2001, Bector and Chandra 2005, Esogbue 1991, Lee and Li 1993, Li and Lee 1990, 1993, Mohan and Nguyen 1998). The collection of papers on the topic of fuzzy optimization edited by Delgado *et al*. (1994) and Slowinski (1998) gave the main stream of this topic. On the other hand, the books by Zimmermann (1996), Lai and Hwang (1992, 1994) also gave the insightful survey.

However, in conventional fuzzy optimization problem the objective functions (say $f_i, i=1,2,..,m$) are expressed as a function of decision variables. In solving practical decision problems, it may be possible that the functional relationship between the decision variables and the objective function is not completely known. We are only able to describe the link between objective functions and decision variables linguistically using fuzzy rule based system. Under these circumstances, a technique needs to be formulated which can transform the fuzzy rule based optimization problem to an equivalent deterministic one (Carlsson and Fuller 2003). In this respect, Carlsson *et al*. (1998), Carlsson and Fuller (2001) addressed fuzzy mathematical programming problems to find a fair optimal solution. In order to explore such an idea into fuzzy multi-objective optimization problems, Carlsson and Fuller (1998) proposed a theory to find a compromise solution to the fuzzy multi-objective mathematical programming problem under fuzzy rule base.

It may be said from the literature that though some theoretical analysis have been made for solving multi-objective optimization problems under fuzzy rules, there is a need to explore the applicability of this idea for solving real-world problems. In view of this, a fuzzy rule based system has been developed in Chapter 8, modeling the linguistic dependencies between the decision variables and the objectives in an improved manner. Subsequently the mechanism, which aims at solving a multi-objective programming problem depending on the given fuzzy rule base, has been illustrated.

1.7.5 Contributions over fuzzy regression analysis

Chapter 9 contains some issues in fuzzy regression paradigm. The classical regression analysis deals with the precise data where the fuzzy regression analysis (e.g., Peters 1994, Tanaka *et al*. 1982 and 1995, Tanaka and Lee 1999) has studied to model linguistic information. Tanaka *et al*. (1982) first proposed fuzzy regression technique in which relation of the variables is subject to fuzziness and a fuzzy linear system was used as a regression model. Also Peters (1994) and Tanaka *et al*. (1995) revised these fuzzy models. Sakawa and Yano (1992) proposed a modified form of possibilistic regression. It should be mentioned that possibilistic regression usually leads to a mathematical programming problem. Diamond (1987, 1988) proposed models of least squares fitting for crisp input and fuzzy output and fuzzy input output. Diamond and Korner (1997) revised the method of least square fitting to fuzzy data. Fuzzy prediction using regression models was studied by Yager (1982). Jajuga (1986) presented a fuzzy approach that is useful for heterogeneous observations. Celmins (1987) dealt with quadratic membership functions to fit indicators of discord, data spread dilator etc. Xu (1997) discussed the problem of least squares fitting of fuzzy valued data by developing S-shaped curve regression model**.** Nather and Korner (1998) extended classical estimates with crisp/fuzzy input and fuzzy output using least-square approach. Wang and Tsaur (2000) considered a type of problem with crisp input and fuzzy output described by Tanaka *et al*. (1982) and it was then solved by a modified fuzzy least square method. Also Durso and Gastaldi (2000) proposed a doubly linear adaptive fuzzy regression model using two linear models: a core regression model and spread regression model. Nasrabadi *et al*. (2005) developed a multi-objective fuzzy linear regression model. Nasrabadi and Nasrabadi (2004), Toyoura *et al*. (2000), Tseng and Lin (2005) reported successful efforts in using regression techniques in the field of engineering, dental and finance, respectively. Fuzzy regression method has also been successfully applied in forecasting (Heshmaty and Kandel 1985, Khemchandani *et al*. 2009). Chakraborty and Chakraborty (2008) developed an approach to fuzzy linear regression modeling for linguistic variables based on a fuzzy rule base. Lu and Wang (2009) proposed an enhanced fuzzy linear regression model in which the spreads of the estimated dependent variables are able to fit the spreads of the observed dependent variables.

It may be said from the review of literature presented above that although researchers have made several suggestions concerning the methodologies and applications of the fuzzy approach to regression analysis, much however, remains to be studied. In view of this, Chapter 9 deals with a linear curve fitting problem under fuzzy environment, giving importance to the satisfaction level of the decision maker.

As evident from the above discussion, though several advancements have been made regarding the development of decision making models under the fuzzy framework, a lot of scope still remains for further development in this regard. With this point of view, in the thesis some fuzzy reasoning schemes are being proposed to deal with different kind of decision models in imprecise environment. The objectives of the thesis are listed below.

- Developing arithmetic operators on GFNs
- Proposing a new concept of fuzzy similarity measure for GFNs
- Developing MADM schemes with single and multiple experts considering the degrees of confidence of the experts
- Estimating priority-based weights of the alternatives under intuitionistic fuzzy environment by employing the concept of similarity measure
- Deriving a distance measure for IFNs
- Solving multi-objective optimization problem under fuzzy rule constraints

- Developing fuzzy linear regression model in a linguistic framework
- Treating different types of real decision problems using the proposed methodologies

With these above objectives, the thesis has been organized in the following manner.

1.8 Organization of the thesis

The thesis has been organized as follows:

The first chapter has given an introduction to the research work along with the motivation and objective.

In Chapter 2, different arithmetic operators on generalized trapezoidal and triangular fuzzy numbers have been derived by employing *extension principle* (Zadeh 1975). In this respect, the shortcomings of the existing operators of GFNs have also been analyzed.

In Chapter 3, a fuzzy distance measure between two GFNs has been developed. A new concept of similarity measure has also been introduced with help of the fuzzy distance measure. Examples have been provided to compare the proposed fuzzy similarity measure with the other existing similarity measures.

In order to solve multiple attribute single expert decision making problems, Chapter 4 presents a fuzzy compromise ratio (FCR) methodology considering the degrees of confidence of expert's opinions. An illustration of the application of the proposed method to a real-life problem has also been presented.

In Chapter 5, a fuzzy multiple attribute group decision making (MAGDM) technique has been developed using GFNs to reach consensus among the linguistic opinions/assessments provided by a group of experts. The performance of the diagnostic laboratory has also been assessed in this chapter, by employing the proposed technique.

A new method for estimating the priority-based weights of alternatives from intuitionistic preference relation has been presented in Chapter 6. For this purpose, a similarity measure of IFSs has also been developed. A numerical example has been presented to illustrate the proposed methodology.

The objective of Chapter 7 is to introduce a methodology to measure the distance of IFNs based on interval difference. This chapter also provides some useful results on IFNs. Furthermore, LR-type IFN has been introduced in this chapter, as IFNs of special type. Numerical examples have been presented for applying the proposed distance measure and finally the result has been compared with the existing ones.

In Chapter 8, a fuzzy multi-objective mathematical programming problem has been considered where functional relationship between the decision variables and the objective functions is not completely known to us. It has been assumed that the information source from where some knowledge may be obtained about objective functions consists of a block of fuzzy if-then rules. The focus in this chapter is to solve the multi-objective optimization problem for the above situations. A numerical example has also been presented to illustrate the method.

A fuzzy linear regression model using logistic membership function has been developed in Chapter 9, wherein both the input and output data are fuzzy numbers. The problem of curve fitting, using the concept of fuzzy inequality, evolves into a fuzzy optimization problem. Finally, the values of the regression coefficients have been estimated by solving the resulting fuzzy optimization problem. Numerical example illustrates the methodology.

Finally, Chapter 10 presents the conclusions on the various investigations carried out in the thesis and a brief discussion on future scope and possible extensions of the present work.