Abstract

The present thesis deals with certain problems associated with delta shock waves for quasilinear one-dimensional hyperbolic system of conservation laws which occur in many physical phenomena having practical importance in real life. Our aim is to solve, those hyperbolic systems where nonclassical situation arises, in various approaches like flux approximation method, vanishing viscosity method, vanishing pressure limit method and distributional product method. Also, we study wave interactions and stability of Riemann solution of hyperbolic systems.

First, we consider the strictly hyperbolic system of conservation laws which describes the background flow carrying dust particles and whose Riemann solution contains delta shock wave as well as classical elementary waves. In order to study stability, we consider the linear approximation of flux functions with three parameters. We prove that the solution of the Riemann problem for the approximated system converges to the solution of the original system when the perturbation parameter tends to zero. Next, we study the Riemann problem for hyperbolic system which arises in nonlinear elasticity and gas dynamics. We discuss the existence and uniqueness of the solution of Riemann problem involving delta shock wave by using self-similar vanishing viscosity approach. We proved that delta shock wave is weak*-limit of L^1 solution to some viscous perturbations as the viscosity vanishes. Further, we investigate the limiting behavior of the Riemann solution to the isentropic Euler equations for logarithmic equation of state with the Coulomb-like friction term. The formation of vacuum state and delta shock waves are identified and analyzed when the pressure vanishes. We prove that the Riemann solution of the isentropic Euler equations for logarithmic equation of state with friction term converges to the Riemann solution of the zero-pressure gas dynamics system with a body force when the pressure vanishes if we neglect the virtual velocity in the vacuum region.

Next, we study the interactions between classical elementary waves and delta shock wave in a thin film of a perfectly soluble anti-surfactant solution in the limit of large capillary and P$\acute{e}$clet numbers. The global structure of the perturbed Riemann solutions are constructed and analyzed case by case when delta shock wave is involved. Finally, we study the delta shock wave for zero-pressure gas dynamics system with energy conservation laws in the frame of $\alpha-$solutions defined in the setting of distributional products. By reformulating the system we construct, within a convenient space of distributions, all solutions which include discontinuous solutions and Dirac delta measure. The constructed $\alpha-$solutions coincide with the solution obtained through different methods.

Keywords: Riemann problem; Elementary waves; Delta shock wave; Flux approximation; Vanishing viscosity; Isentropic Euler system; Coulomb-like friction term; Vacuum state; Wave interactions; Anti-surfactant; Nonstrictly hyperbolic system; Product of distributions; Zero-pressure gas dynamics with energy conservation laws