Faraday Instability in Classical and Quantum Fluids

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Faraday waves at the flat free surface of low viscous fluid motivate many scientific communities to study further the Faraday instability in various mediums. Their experimental and theoretical endeavours have fundamental importance in pattern formation, measuring surface tension, etc. Although this instability at the flat free surface is well investigated, it is not well explored at curved free surfaces. Here, we present the results of our experimental investigations on parametrically driven waves in a water half-cylinder on a rigid horizontal plate, which is sinusoidally vibrated in the vertical direction. As the forcing amplitude is raised above a critical value, stationary waves are excited in the half-cylinder water. Parametrically excited subharmonic waves are non-axisymmetric and qualitatively different from the axisymmetric Savart-Plateau-Rayleigh waves in a vertical liquid cylinder or jet. Depending on the driving frequency, stationary waves of different azimuthal wavenumbers are excited. We supplement a linear theory that captures the observed dispersion relations qualitatively. On the other hand, we also observed parametrically excited standing waves on a cylindrical fluid filament. This excitation is cylindrical analogous to the Faraday instability in a flat surface. Using Floquet theory, a linear stability analysis is carried out on a viscous cylindrical fluid surface subjected to a time-periodic radial acceleration. The viscosity of the fluid has a significant impact on the critical forcing amplitude and the dispersion relation of the non-axisymmetric patterns. The threshold of pattern formation with azimuthal wavenumber m=1 shows differently dependent on the viscosity than m>1. Moreover, our study shows that the effect of viscosity is stronger on the threshold for higher m. Therefore, the stable modes in Rayleigh-Plateau instability are unstable under periodic forcing.

We have extended our experience in the quantum fluid. The sharp interface in a strongly phase-separated binary Bose-Einstein condensates (BEC) is ideal for studying Faraday instability in a quantum system. We investigate a Faraday-wave-like parametric instability via mean-field and Floquet analyses in an immiscible binary ultra-cold atomic system. The condensate components form a so-called ball-shell structure in a two-dimensional harmonic trap. To trigger the dynamics, the scattering length of the core condensate is periodically modulated in time. Once the modulation starts in the dynamics, the interface becomes unstable, and oscillating patterns form. The subharmonic oscillations of the interface exhibit an *m*-fold rotational symmetry. The value of *m* can be controlled by manoeuvring the amplitude and the frequency of the modulation. Using Floquet analysis, we can predict the interfacial tension of the binary mixture and derive a dispersion relation for the natural frequencies of the emergent patterns. For the experimental realization of the phenomenon, we would like to propose a heteronuclear condensate composed of ^{87}Rb—^{85}Rb. However, our results are independent of involved atomic species and the applied driving parameter. Pattern forming instability is also investigated via mean-field in a single component Bose-Einstein condensate in a weakly dissipative environment under periodic modulation of scattering length. The finite temperature of the systems is incorporated by the inclusion of the dissipation term in the study. Azimuthal wavenumber (*m*) of the pattern initially increases with frequency up to a certain value. The value of wavenumber fluctuates (increases and then sudden decreases to a minimum value, repeatedly) with increasing frequency. However, along with this process, an additional bulk pattern emerges with the surface pattern in every higher frequency domain. At much higher frequencies, squares, lines, or competing square and lines subharmonic lattice patterns are also observed. Calculating Fourier modes amplitudes of the patterns, we able to describe the non-linear pattern formation in a huge frequency domain.

List of Publications

1. Instability of a horizontal water half-cylinder under vertical vibration, **D. K. Maity**, K. Kumar and S. P. Khastgir, Exp Fluids **61**, 25 (2020).
https://doi.org/10.1007/s00348-019-2860-9

2. Floquet analysis on a viscous cylindrical fluid surface subject to a time-periodic radial acceleration, **D. K. Maity**, Theor. Comput. Fluid Dyn. **35**, 93 (2021).
https://doi.org/10.1007/s00162-020-00550-y

3. Parametrically excited star-shaped patterns at the interface of binary Bose-Einstein condensates, **D. K. Maity**, K. Mukherjee, S. I. Mistakidis, S. Das, P. G. Kevrekidis, S. Majumder and P. Schmelcher, Phys. Rev. A **102**, 033320 (2020).
https://link.aps.org/doi/10.1103/PhysRevA.102.033320

4. Pattern Formation in Weakly dissipative Bose-Einstein condensate, **D. K. Maity**, K. Mukherjee, and S. I. Mistakidis, and S. Das, and P. G. Kevrekidis, and S. Majumder, and P.Schmelcher . (To be submitted)