Thesis Title: Algorithms on Temporal Graphs

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Abstract

Temporal graphs have been used to model systems and applications in many domains where objects and the relations between them may vary with time, resulting in graphs with dynamically changing topologies and/or other properties. Different graph structures such as shortest paths, spanning trees, matchings, vertex covers etc. play an important role in many such applications. However, the definitions and algorithms for constructing many of these graph structures cannot be directly extended from the area of static graphs to the area of temporal graphs because of the dynamically changing topology. In this thesis, we address the definition and construction of three graph structures on temporal graphs, namely convergecast trees, dominating sets, and matchings.

Convergecast trees are useful for collecting data at a central node in many applications with distributed data generation. In a temporal graph, a path or journey from a source node to a destination node is said to exist if there exists a sequence of edges from the source to the destination that occur in strictly increasing order of time. A convergecast tree rooted at a node in a temporal graph is a minimum sized subgraph of the temporal graph such that there exists exactly one journey from each non-root node to the root node. In the first work of this thesis, we define two types of convergecast trees on temporal graphs, namely Bounded Arrival Time Convergecast Tree (BATCT) and Minimum Total Arrival Time Convergecast Tree (MTATCT). We first propose an algorithm for constructing a BATCT. We then show that the problem of constructing a MTATCT is NP-Complete, and propose and evaluate a heuristic algorithm for it.

Dominating sets have been used for applications such as information dissemination in many systems. A permanent dominating set in a temporal graph is a subset of nodes such that any node in the graph at any timestep either belongs to the permanent dominating set or is adjacent to some node in the permanent dominating set. In the second work of this thesis, we study the problem of finding a permanent dominating set on temporal graphs. We first show that the problem of constructing a minimum permanent dominating set is NP-Complete, and propose and evaluate an approximation algorithm for it. We also define a structure called maximum k-dominant node set for a given temporal graph. We prove that the problem of constructing a maximum k-dominant node set is also NP-Complete and propose an approximation algorithm for it.

In the third work of this thesis, we study the problem of constructing matchings on temporal graphs. Matchings are used in many types of assignment problems. We define a type of matching called 0-1 timed matching on temporal graphs, which is defined as a subset of edges such that no two edges are incident on the same node at the same timestep. We study the complexity of constructing a maximum 0-1 timed matching on different classes of temporal graphs. In particular, we show that the problem is NP-Complete for a class of rooted temporal trees, but is solvable in polynomial time for a subclass of it. We then show that the problem is NP-Complete for temporal graphs in general, and remains NP-Complete for bipartite temporal graphs and degree bounded temporal graphs. Finally, we propose an approximation algorithm for constructing a maximum 0-1 timed matching on temporal graphs.