Abstract

This thesis addresses one point and multipoint higher order iterative methods, their convergence analysis and dynamics for solving nonlinear equations in Banach spaces. Developing efficient iterative methods is one of the most important and challenging tasks in scientific computing and numerical functional analysis. The solutions of large number of applications require solving thousands of such equations in one or several parameters. The thesis starts by developing a family of parameter based higher order iterative methods free from second derivative for $f(x) = 0$, where $f : \mathbb{R} \to \mathbb{R}$ and investigates their theoretical, computational and dynamical aspects. It includes sixth order methods and for a particular value of parameter leads an eighth order method. Both of them require three functions and one first derivative evaluations. Local convergence analysis and dynamics are also carried out. The local convergence of a family of iterative methods based on a parameter θ for multiple roots with known multiplicity m is established under the assumption that the derivative $f^{(m+1)}$ of function f satisfies the Hölder continuity condition. The R-order of convergence is shown to be equal to $(2 + p)$, where $p \in (0, 1]$. The well known methods of Chebyshev $(\theta = 0)$ and of Osada $(\theta = 1)$ belong to the family. The radii of convergence balls for different values of parameter θ are compared and tabulated. Moreover, its dynamical study is fully investigated and parameter of plane is constructed to find the best value of the parameter θ. The values of θ are investigated according to character of the strange fixed point and the multiplicity of the root. By using majorizing sequences, the semilocal and local convergence analysis for two-step Secant method to approximate a locally unique solution of a nonlinear equation in Banach spaces is established under the assumption that the first order divided differences and the Fréchet derivative of the involved operator satisfy the weaker Lipschitz and the center-Lipschitz continuity conditions. As a result, finer majorizing sequences and enlargement of convergence domain of the solution are found. Also, by taking a nonlinear system of equations, the Efficiency Index (EI) and the Computational Efficiency Index (CEI) of two-step Secant method are computed and its comparison with respect to other similar existing iterative methods are summarized in tabular forms. The semilocal and local convergence analysis of a two-step iterative method for nonlinear nondifferentiable operators are also described in Banach spaces. The recurrence relations are derived under ω-continuity conditions. For semilocal convergence, the domain of parameters are obtained to ensure guaranteed convergence under suitable initial approximations. Similarly, we enhance the region of accessibility and applicability for local convergence. This provides a way to enlarge its convergence domain also. The convergence and dynamics of improved Chebyshev-Secant-type iterative methods are studied. Its semilocal convergence is established using recurrence relations under ω-continuity conditions on first

order divided differences. The center-Lipschitz condition is defined on the first order divided differences and its influence on the domain of starting iterates are compared with the domain of Lipschitz conditions. The dynamical analysis of the iterative method is carried out. It confirms better stability properties than its competitors. The directional k-step Newton methods (k a positive integer) is developed for solving a single nonlinear equation in n variables. Its semilocal convergence analysis is established by using two different approaches (recurrent relations and recurrent functions) under the assumption that the first Fréchet derivative satisfies a combination of the Lipschitz and the center-Lipschitz continuity conditions instead of only Lipschitz continuity condition. It is shown that the second approach based on recurrent functions solves problems failed by using recurrent relations. This work extends the directional one and two-step Newton methods for solving a single nonlinear equation in n variables. The computational order of convergence and the computational efficiency are also provided. By using majorizing sequences and weaker Lipschitz type bound operators, the semilocal and local convergence of Secant-like methods are described for solving nonlinear operators in generalized Banach spaces. Newton's method, Secant method and many other similar iterative methods are particular cases of it. The information about the precise location of the solution can also be estimated. The local convergence analysis of deformed Super Halley's method under weaker continuity conditions on first order Fréchet derivative is established. This work generalizes the earlier work in this direction and it is observed that it is applicable to cases where either they fail to converge or give smaller balls of convergence. The semilocal convergence of an efficient fifth order iterative method is established under weaker conditions for solving nonlinear equations. It is done by assuming ω-continuity condition on second order Fréchet derivative. The novelty of our work lies in the fact that several examples are available where Lipschitz and Hölder continuity condition fails but ω-continuity condition holds. The R-order is found to be $4+q$, $q \in (0,1]$.

For all the above mentioned higher order iterative methods, theorems are established for the existence-uniqueness regions along with the estimation of a priori and a posteriori error bounds on the solutions. By using MATLAB R2012b on an Intel(R) Core (TM) i5-3470 CPU 3.20GHz with 4GB of RAM running on the Windows 7 Professional version 2009 Service Pack 1 , a number of numerical examples including Automotive Steering problems, nonlinear mixed Hammerstein type integral equations, nonlinear elliptic differential equations and integral equations are worked out to demonstrate their efficiency and applicability. The results obtained are compared with those obtained by some of the existing similar higher order iterative methods.