Abstract

This thesis mainly focuses on the study of some basic subclasses of harmonic functions in the unit disk $\mathbb{D}$. The thesis contains an application of conformal mappings, which solves a two dimensional potential flow problem in an anisotropic domain.

Chapter 1 of the thesis is introductory.

Chapter 2 and **Chapter** 3 concerns a study of new subclasses of harmonic functions. After introducing these new subclasses, we prove that functions in these subclasses are univalent in $\mathbb{D}$, and are harmonic analogues of well known geometric subclasses of univalent analytic functions. Next, we investigate some basic properties of functions in these new harmonic subclasses, again in relation to geometric function theory. In particular, we determine sharp coefficient bounds, and growth theorems for functions in these classes. We also study the convex combination, and convolution properties for functions in these subclasses. We discuss the two points distortion theorem, and as a consequence study the sections of univalence for functions in these subclasses. Moreover we construct harmonic univalent polynomials by considering the co-analytic part of harmonic functions as Gaussian hypergeometric functions.

In **Chapter** 4 we primarily solve radius problems of harmonic functions. More preciously we determine the radius of fully starlikeness, and the radius of fully convexity for a harmonic operator under certain coefficient conditions. We also determine the radius of uniformly starlikeness, and the radius of uniformly convexity of harmonic functions.

In **Chapter** 5 we give an application of conformal mappings to a fluid flow problem. In particular, we present an analytical solution to a two-dimensional flow to a well in an anisotropic domain with hydraulic conductivities varying along all directions. We derive the solution using a suitable coordinate transformation, and using conformal maps to meet the constant head condition at the boundary of the well.

Keywords: Analytic, harmonic, conformal, univalent, starlike, convex, close-to-convex, Gaussian hypergeometric functions, subordination, coefficient bounds, growth theorem, radius of univalence, convolution, anisotropic, hydraulic conductivity, complex potential, discharge potential, stream function.