Abstract

Spectral graph theory helps to characterize some principal and structural properties of a graph from the associated graph spectra. There are several kinds of spectra associated with a graph; adjacency, Laplacian, and normalized Laplacian to name a few. In this thesis, we mainly study the spectrum of normalized Laplacian matrix of a simple graph, where the matrix is defined as $\mathcal{L} = I - D^{-1/2}AD^{-1/2}$, D is the diagonal matrix of vertex degrees and A is the adjacency matrix of the graph. The graphs for which we compute normalized Laplacian spectrum are obtained by several graph operations, broadly, coronas, subdivision-coronas, subdivision-joins, subdivision-subdivision joins, R-coronas, R-joins, R-R joins, subdivision-R joins, Q-coronas, and Q-joins. Some of these operations were defined by previous authors and the operations like subdivision-vertex-vertex join, subdivision-edge-edge join, subdivision-edge-vertex join, R-vertex-vertex join, R-edge-edge join, R-edge-vertex join, subdivision-vertex-R-vertex join, subdivision-edge-R-edge join, subdivision-edge-R-vertex join, subdivision-vertex-R-edge join, Q-vertex join, and Q-edge join are introduced in the thesis. An important application of these graphs is representation of complex networks in terms of the smaller ones. The main technique followed in the thesis to find normalized Laplacian spectrum of graphs is that we formulate normalized Laplacian matrices of graphs in terms of Kronecker product and Hadamard product, and then use some linear algebraic properties including Schur complement. Some graphs with five distinct normalized Laplacian eigenvalues are also obtained in the thesis.

Applying all the graph operations discussed in the thesis we construct non-regular nonisomorphic simultaneous cospectral graphs with respect to adjacency, Laplacian and normalized Laplacian matrices. Thus we give a answer to the question "Is there an example of two non-regular graphs which are cospectral with respect to the adjacency, Laplacian and normalized Laplacian at the same time?" asked by Butler (2010). Characterizing a graph theoretic property is an important and fundamental result. We prove that every n-vertex simple connected graph with independence number $\alpha > \frac{n}{2}$ has 1 as a normalized Laplacian eigenvalue with multiplicity at least $2\alpha - n$. Then a natural question arises "What are all the n-vertex graphs with independence number less than or equal to $\frac{n}{2}$ and that have 1 as a normalized Laplacian eigenvalue?". We investigate this question upto some extent in the thesis.

Keywords: Graphs; Normalized Laplacian matrix; Schur Complement; Normalized Laplacian spectrum; Adjacency spectrum; Laplacian spectrum; Coronas; Subdivision-coronas; Subdivision-joins; Subdivision-subdivision joins; R-coronas; R-joins; R-R joins; Subdivision-R joins; Q-coronas; Q-joins; Kronecker product; Hadamard product; Cospectral graphs; Simultaneous cospectral graphs; Independence number.