Abstract

The present thesis deals with some problems associated with univalent functions defined either on the unit disk in the complex plane or on the exterior to it, that have quasiconformal extension onto the extended complex plane. Our main concern is to study meromorphic univalent functions defined on the unit disk with a pole at some nonzero point $p \in (0,1)$ that have k-quasiconformal extension onto the extended complex plane. We denote the class of such functions by $\Sigma_k(p)$. At first, we prove an area theorem for functions in $\Sigma_k(p)$. We also prove a sufficient condition for meromorphic univalent functions with nonzero pole to belong to this class. Using this, we obtain the following result: if $f \in \Sigma_{k_1}(p)$ and $g \in \Sigma_{k_2}(p)$, then we find $\alpha \in (0,1)$ for which the Hadamard product $f \star g \in \Sigma_\alpha(p)$.

Next, we obtain estimates for the n-th Laurent coefficients of functions in $\Sigma_k(p)$ and also we derive a distortion result for functions belonging to this class. The obtained results generalize the corresponding results for meromorphic functions with zero pole that have k-quasiconformal extension onto the whole complex plane.

Further, we are interested in answering the following question: how area of a Lebesgue measurable set distorts under a function in the class $\Sigma_k(p)$? In doing so we prove a sharp area distortion and a sharp weighted area distortion inequality for this class. As an application, we give a sharp bound for the Hilbert transformation of characteristic function of a Lebesgue measurable set.

Finally, we prove quasiconformal extendibility of some classes of univalent functions. We also find some criteria for quasiconformal extension of meromorphic functions with pole at the origin. Some of the aforementioned quasiconformal extensions have been obtained by constructing suitable Loewner chains and others have been obtained by applying a well-known result.

Keywords: Analytic, meromorphic, univalent, conformal map, quasiconformal map, quasiconformal extension, Area theorem, convolution, distortion estimate, coefficient estimate, area distortion, Hilbert transformation, Loewner chain.