Abstract

This thesis addresses the computational methods to compute eigenvalues and generalized inverses of real and complex matrices and the application of interval analysis to derive rigorous error bounds for them. Development of efficient computational methods for handling eigenvalues and generalized inverses of real and complex matrices and their associated errors are most important and challenging problems in applied mathematics and numerical analysis. Chapter 3 describes two approaches to establish verifiable sufficient regularity conditions of complex interval matrices used to compute the bounds of interval eigenvalues sets. In the first approach, a complex interval matrix is mapped to a real block interval matrix, and then its sufficient regularity conditions are obtained. In the second approach, a necessary condition for the singularity of a complex interval matrix is derived and used to get its sufficient regularity conditions. As an application, the above derived sufficient regularity conditions are used to investigate the location of the outer approximations of individual eigenvalue sets of complex interval matrices. Two algorithms are proposed, and results obtained are compared with those obtained by earlier methods and Monte Carlo simulations. The advantages of these algorithms are that they can detect gaps in between the approximations of the whole eigenvalue sets much better than the existing methods. The second algorithm is very effective compared to the first algorithm regarding computational time. Several numerical examples and statistical experiments are worked out to validate and demonstrate the efficacy of our work. The novelty of this work is to achieve gaps in between the outer approximations of whole eigenvalues sets and to find outer approximations for individual eigenvalue sets of real or complex interval matrices using their sufficient regularity conditions. In Chapter 4, an iterative method is proposed to compute the outer eigenvalue bounds for the real symmetric interval matrices. It uses Jordan canonical form of real symmetric interval matrices to approximate their eigenvalue bounds. First, it obtains a midpoint matrix of a real symmetric interval matrix into its Jordan canonical form and then constructs an interval matrix whose midpoint matrix is diagonal such that its singularity and eigenvalue bounds are preserved. Next, the eigenvalue bounds are computed using a sufficient regularity condition. The algorithm and corresponding computational costs are given. The benefits of this work are that it substantially reduces the operations involved in the computation of inverses of real matrices. The efficiency and applicability of the iterative method are demonstrated by working out five numerical examples. One of them considers randomly generated large dimensional interval matrices. The dependency of the bounds by the radius of the input matrix are shown in the tabular form. It is observed that the eigenvalue bounds are improved if the radius of the symmetric interval matrix is small. Obtained results are comparable to those given earlier. In Chapter 5, the diagonal stability and generalized diagonal stability for three classes of parametric interval matrices are considered. Necessary and sufficient conditions for the Schur diagonal stability (SDS_p) and Hurwitz diagonal stability (HDS_p) of a real matrix under Hölder norm which generalizes the definitions of Schur and Hurwitz stability are developed. Some verifiable sufficient conditions for SDS_p and HDS_p of the first parametric class of matrices are given using majorant matrices. Next, using another set of majorant matrices, necessary and sufficient conditions for SDS_p and HDS_p are obtained for the second and third parametric classes of matrices, respectively. It is found that SDS_2 / HDS_2 for complex interval matrices are its special cases. Methods for finding common diagonal matrix satisfying the SDS_p / HDS_p conditions are given. The robustness analysis of SDS_p / HDS_p is also carried out. Chapter 6 describes a new iterative scheme for deriving error bounds of the Moore-Penrose generalized inverse $A^\dagger$ of an arbitrary rectangular or singular real or complex matrix A of arbitrary rank. An approach based on a sequence of inclusion interval matrices containing the Moore-Penrose inverse is used. The proposed iterative scheme requires that the rank of A is known or computed. Starting from the initial interval matrix constructed appropriately by using the initial approximation to the Moore-Penrose inverse in conjunction with the hyperpower iterative method for generating successive approximations, a sequence of monotonic inclusion interval matrices is generated. Each interval matrix included in the sequence contains $A^\dagger$. A convergence theorem is established. Theoretical results show that the sequence of monotonic inclusion interval matrices converges to $A^\dagger$. Numerical examples involving randomly generated matrices are solved to demonstrate the efficacy of our approach. In comparison with the results obtained using the method from Zhang et al. (2006), it is found that our approach gives better accuracy. In Chapter 7, an interval extension of SMS method called ISMS method for computing weighted Moore-Penrose inverse $A^\dagger_{MN}$ for full row or column rank complex rectangular matrices A, for Hermitian positive definite matrices M and N, along with its guaranteed rigorous error bounds are proposed. The classical floating point numerical computation of SMS method always provides an approximation to the weighted Moore-Penrose inverse, not the exact one. Starting with a suitably chosen complex interval matrix containing $A^\dagger_{MN}$, the method generates a sequence of complex interval matrices each enclosing $A^\dagger_{MN}$ and converging to it. A new method is developed for constructing an initial complex interval matrix containing $A^\dagger_{MN}$. Convergence theorems are established. The R-order convergence is shown to be equal to at least l, where $l \geq 2$. Some numerical examples are worked out to demonstrate its efficiency and effectiveness. Graphs are plotted to show variations of the number of iterations and computational times compared to matrix dimensions. It is observed that ISMS method is more stable compared to SMS method. In Chapter 8, a quadratically convergent iterative method based on Newton's method is developed for approximating Drazin inverse b^d of b in Banach algebra $\mathscr{J}$. It is based on an iterative method developed by Srivastava and Gupta (2015) for approximating the generalized outer inverse $A^{(2)}_{T,S}$ of $A \in \mathscr{B}(\mathscr{X},\mathscr{Y})$, where $\mathscr{B}(\mathscr{X},\mathscr{Y})$ denotes the set of bounded linear operators between Banach spaces $\mathscr{X}$ and $\mathscr{Y}$. Convergence results of improved approximate solutions, as well as the error estimates, are derived. Further, its extension to a kind of the hyperpower iterative method is also described, and the results concerning its convergence and the error estimate are established.