Abstract

Fractional calculus is an interdisciplinary research subject, which originated with the idea of introducing non-integer order derivatives (or integrals) and exploring its applications to various branches of applied sciences and engineering. Recently, it has captured the attention of researchers by finding significant applications in various fields such as optimal control theory, signal processing, viscoelasticity, anomalous transport and diffusion models, etc. The apparent modernization of classical problems in the sense of fractional calculus inspires researchers to look for admissible applications and physical properties unattended by integer order operators. Despite its limited literature as compared to the classical calculus, the research on fractional calculus has shown very rapid interest in several applications such as fractional calculus of variations, and fractional optimal control problems, etc.

The thesis can be partitioned into three segments. In the first part (Chapter 2-4), a class of generalized fractional variational problem is presented, which is modeled with classical and fractional order operators. The suggested problem exhibits extension of classical as well as fractional calculus of variations. The prime motive of this work is to organize the proposed generalization to its maximum extent. From the theoretical point of view, we have established first and second order necessary optimality conditions. The first order necessary optimality condition is expressed as a fractional integro-differential equation, termed as fractional Euler-Lagrange equation. Whereas, second order condition takes the advantage of second variation of a concerned functional to develop Jacobi and Legendre variational tests. We have further strengthened the Legendre variational test to deduce the sufficient optimality conditions. This work conjointly presents a numerical scheme to approximate compositions of fractional order operators, which shows that a solution of posed fractional variational problem can also be obtained by approximating a composition of left and right fractional derivatives occurring in fractional Euler-Lagrange equations. In addition to this, examples demonstrating the formulations have been provided with sufficient numerical information.

The second part of the thesis (Chapter 5-6) is devoted to investigate distinct classes of fractional optimal control problems, which comprises of fractional differential or integro-differential equations as system dynamical constraints. After examining the structure of previously studied problems, we have presented a generalized class of fractional optimal control problem. The proposed generalization arises due to the possibility to extend the fractional operators terms in the governing equations of the system. We have established the necessary and sufficient optimality conditions for each of these classes. This work also accommodates direct and indirect numerical algorithms to acquire solutions of fractional optimal control problems. Namely, Adomian decomposition method to solve necessary optimality conditions which is considered to be an indirect scheme. Consequently, we have designed a well-organized direct numerical scheme to optimize the given problem exercising Laguerre orthogonal polynomials and Boubaker non-orthogonal polynomials, independently. The key motive associated with the presented approach is to convert the concerned fractional optimal control problem to an equivalent quadratic pro-

gramming problem with linear equality constraints, which can be handled efficiently. We have worked out few examples to illustrate the computational technique along with its efficiency and accuracy. Graphical representations are also provided to analyze the performance of the state and control variables for distinct prescribed fractions.

In the last part (Chapter 7), we have introduced a new concept of α-fractionally convex function. The idea to explore convex functions with fractional order derivatives parallel to classical convexity seems to be intuitive. We have provided enough motivation and well-founded reasons in support of the suggested formulation. The prime focus is to characterize essential properties and results of fractionally convex functions along with some examples. We have also investigated possible connections between critical points, optimality, monotonicity and convexity of a function in the sense of fractional calculus. The thesis finishes in Chapter 8 with a conclusion and scope of future works of the covered aspects of fractionally convex functions, fractional problems of the calculus of variations and optimal control.

Keywords: Riemann-Liouville fractional derivative, Riemann-Liouville fractional integral, Caputo fractional derivative, Euler-Lagrange equation, Adomian decomposition method, Laguerre polynomial, Baubaker polynomial, Quadratic programming problem, Convergence analysis, Jacobi condition, Legendre condition, Optimal value, monotonic function, critical point, Convex function.