

Automatic Determination of the Neutral Curve of Linear Stability of Steady and Time-Periodic Flows using Pseudo-Arclength Continuation

Abstract

A procedure for accurate and automatic determination of the neutral curves of linear stability of steady and time-periodic parallel flows of constant-density and thermally stratified fluids has been developed. The differential equations of linear stability are discretized using a spectral collocation technique which yields infinite order of accuracy or exponential convergence. The eigenvalues of the resulting generalized eigenvalue problem are determined using the QZ algorithm. The leading eigenmode which represents the most unstable or least stable disturbance is identified by sorting the eigenvalues. The locus of points representing the neutrally stable leading modes in the stability parameter space is traced using pseudo-arclength continuation. Methods based on continuation in a natural parameter may fail near the turning points of the curves. This difficulty is circumvented by parametrizing the curves using a pseudo-arclength parameter. Use of a global method for determining the eigenvalues ensures that the leading mode is not missed. A direct method for accurate determination of the critical parameters for onset of instability has also been proposed. The methods have been illustrated by computing the marginal curves of linear stability of plane Poiseuille, Couette-Poiseuille, unstably stratified Couette-Poiseuille and stably stratified Poiseuille flows and Rayleigh-Benard convection. The computed values of the critical parameters are more accurate than previously reported values, for all the test problems except Rayleigh-Benard convection.

The proposed methods are used to investigate the stability of stably stratified Couette-Poiseuille flow, a study not reported before. Marginal stability curves and the corresponding critical parameters have been computed for a wide range of values of the Richardson number and the ratio of the velocity of the upper plate to the maximum Poiseuille velocity. The results demonstrate that stable thermal stratification has a stabilizing influence on the Tollmien-Schlichting shear instability.

The proposed methods are also used to study the stability of time-periodically modulated plane Poiseuille flow in a parameter range not considered before. It has been shown that for certain ranges of amplitude and frequency parameters, the neutral stability curves have multiple disconnected branches. Some of the branches appear as closed curves in the stability parameter space of Reynolds number and wavenumber.

Keywords: Linear Stability, Neutral curve, Pseudo-arclength Continuation, Time-periodic Flows, Stratified Flows, Orr-Sommerfeld Equation, Floquet Theory, Richardson Extrapolation, Heinrichs Basis, Trapezoidal rule, Chebyshev Spectral method