Projection Methods for Integral Equations

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13MA90J12

Abstract

In this thesis, we consider the numerical approximation of linear and nonlinear Volterra and Fredholm-Hammerstein integral equations using piecewise and Legendre polynomial based projection methods. We develop the re-iteration techniques for Galerkin and collocation methods for linear Volterra integral equations of the second kind with a smooth kernel using piecewise constant functions and obtain improved convergence rates for every step of iteration. We also consider the piecewise polynomial based Galerkin and multi-Galerkin methods to approximate the solutions of linear Volterra and nonlinear Volterra integral equations of Urysohn type with a smooth kernel. We obtain superconvergence results for the iterated approximate solutions of Galerkin and multi-Galerkin methods in infinity norm. To get better accuracy in piecewise polynomial based projection methods, one has to solve a large system of equations because of the large number of partition points. To avoid this, we consider the Legendre Galerkin, Legendre collocation and Legendre multi-Galerkin methods to approximate the solutions of linear Volterra integral equations of second kind with a smooth kernel and obtain the superconvergence results in iterated Legendre Galerkin, iterated Legendre collocation and iterated Legendre multi-Galerkin methods in both infinity and L^2-norm. Next we develop the Legendre multi-projection methods for solving the Fredholm-Hammerstein integral equations of second kind with a smooth kernel and establish improved superconvergence results for the approximate and iterated approximate solutions in Legendre multi-Galerkin and Legendre multi-collocation methods in both infinity and L^2-norm.