Abstract

Spectral theory of bounded linear operators occupies an important position in modern functional analysis due to its several applications in various fields. In Banach space, the spectrum of a bounded linear operator is classified in many ways. One of the important classification among them is the point spectrum, the continuous spectrum and the residual spectrum. Recently, an important classification of spectrum is given by Goldberg (1966). Many mathematicians such as Rhoades, Reade, Gonzalez, Altay, Basar, Srivastava and many others have shown their interest in this direction to get the spectrum and the fine spectrum of some classes of bounded linear operators. In order to locate the spectrum of a bounded linear operator on Hilbert space, numerical range of the bounded linear operator plays an important role.

The study of geometrical properties of the numerical range of a bounded linear operator provides a powerful tool to understand the operator as well as its spectrum. Two great mathematician Toeplitz and Hausdorff have shown that the numerical range of a bounded linear operator is always a convex set in the complex plane. Using this fact, many mathematicians such as Donoghue, Hildebrandt, Sims, Shields, Lancaster, Ridge, Chien, Spitkovsky, and many others have derived interesting results regarding the numerical range of bounded linear operator on Hilbert space. The present work is aimed to obtain some spectral results of triangular band matrices with finite bandwidth over some sequence spaces and to study the geometrical properties of the numerical range of some classes of perturbed operators on Hilbert space. The thesis consists of six chapters including introductory chapter.

Chapter 1 deals with motivation, survey of earlier research work carried out in the direction of the spectrum of linear operators in both Banach and Hilbert space and also the works done on the shape of the numerical range of bounded linear operators on Hilbert space. Relevant definitions and known results regarding spectrum and numerical range are also discussed.

Chapter 2 is devoted to study the spectrum, the point spectrum, the continuous spectrum and the residual spectrum of some operators which are of the form $P_n(T)$, where $P_n(z)$ denotes the polynomial of degree n and T is a bounded linear operator acting on a Banach space. Using these results, we derive the results on the spectrum, the point spectrum, the continuous spectrum and the residual spectrum for the infinite upper and lower triangular band matrix with finite bandwidth, acting on some sequence spaces. These results unify the results of earlier workers and can be obtained as particular case on choosing a suitable type of $P_n(z)$.

In Chapter 3, the spectrum, the point spectrum, the continuous spectrum and the residual spectrum along with some Goldberg's classification of spectrum for the operator $\Delta_{a,b}$ on

the sequence space $\ell_p(1 < p < \infty)$ are also studied, where the sequences $a = \{a_n\}$ and $b = \{b_n\}$ are not necessarily convergent.
Chapter 4 deals with some conditions under which the boundary of the numerical range of the Jacobi operator J does not have any point spectrum. Using these results we have also discussed the possible shape of the boundary of the numerical range for the operator J.
In Chapter 5, Anderson's theorem are generalized for the operator which is of the form $T = N + K$, where N is a normal operator and K is a compact operator. The possible shape of the numerical range of the operator $T = H_0 + K$, where H_0 is a Hermitian operator and K is a compact operator are also discussed.
Chapter 6 highlights the conclusion as well as future scope of research in the direction of the work done in the thesis.

Keywords: Difference operator; Shift operator; Band matrices; Spectrum of an operator; Fine spectrum; Sequence spaces; Infinite matrices; Numerical range; Normal operator; Compact operator; Weighted shift operator; Bounded linear operator.